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ATTENTION.[1]

AFFECTIVE CONFLICT AND UNITY OF CONSCIOUSNESS.

ALTHOUGH attention may boast of possessing more abundant literature than any other psychical phenomenon, yet it is still far from being fully explained; that is to say, it has not been brought to any extent into relation and association with other psychic phenomena, especially with those to which it is most closely related. And although attention, as Titchener rightly emphasizes, forms the very pivot upon which all psychology hinges, yet to-day the question as to its inmost nature is still very far from solution. What a great loss this branch of science suffers thereby it is easy to conceive.

The cause of this delinquency in the scientific explanation of attention holds true also for all other psychic activities, namely, that the investigation of all these phenomena has been begun at just the point where they are the most complex and intricate instead of beginning with the simplest forms. The question of attention has usually been taken up by means of self-contemplation and at the moment of philosophical reflection, instead of by observing, for instance, the beast of prey, impatient to fall upon the quarry he has espied and for which he has long lain in wait, or the child who would fain put a white pellet in his mouth but is in doubt whether it is a piece of candy as usual, or may turn out to be a bitter pill as was yesterday the case.

[1] Translated from the German which is to appear in the *Archiv für Psychologie.*

The expediency of beginning the investigation with the simplest forms involves the expediency of pursuing the phylogenetic method and following the course of evolution back as far as possible in order to reveal the phenomenon in the very moment of its first appearance. This is the course we pursued when investigating the inmost nature of another psychic phenomenon no less important and fundamental, namely that of affective tendencies, and the phylogenetic research which showed us their mnemonic origin and nature at once threw light upon that class of phenomena previously so obscure.[2]

We believe that this procedure will attain the same success in our study of attention, which however as we shall see is only a secondary phenomenon directly derived from affective tendencies.

In the treatise just mentioned, "On the Mnemonic Origin and Nature of Affective Tendencies," we have seen that these tendencies are originally only expressions of one and the same intrinsic tendency of the organism to preserve or restore the state of its physiological equilibrium, or to reestablish a previous physiological state which had been determined in the past by certain environmental relations. As soon as these relations are even partially repeated they bring about the "discharge" of the mnemonic accumulation which this former physiological system had left behind.

Then from these affective tendencies of direct mnemonic origin which strive to reestablish certain environmental relations as a whole, arise, according to the known law of affective transference of the whole to the

[2] E. Rignano, "Dell' origine e natura mnemonica delle tendenze affettive," *Scientia*, No. XVII, 1, 1911; "Ueber die mnemonische Entstehung und die mnemonische Natur affektiver Neigungen," *Archiv für die gesamte Psychologie*, Vol. XX, No. 1, 1911; "On the Mnemonic Origin and Nature of Affective Tendencies," *Monist*, July, 1911. This treatise later appeared also as Appendix to the English edition of the author's work, *On the Inheritance of Acquired Characters; An Hypothesis of Heredity, Development and Assimilation.* Chicago, The Open Court Publishing Company, 1911.

part, all the other affective tendencies of indirect mnemonic origin which strive to reestablish only very definite parts or details of these environmental relations. Besides the most important environmental relations usually striven for eagerly in their original totality, the higher animals, and especially mankind, always possess a large number of secondary and even quite specific, environmental relations which in this way are capable of becoming in their turn objects of desire.

At this point we must emphasize the fact that when a physiological system has been disturbed by altered environmental conditions and reduced to a potential state in the form of a mnemonic accumulation, it can become fully reactivated and continue active in a stable physiological state only when its internal and external relations are entirely and exactly the same as when they induced this physiological state. Thus the physiological system of an infusorian which has previously lived in a certain temperature or in a salt solution of a certain proportion will generate an affective tendency toward return to its former habitat as soon as it is removed to other environmental relations; and this tendency will be expressed by negative reactions to every other change of its environmental relations which tends to remove it still further from its original habitat, and by positive reactions to every change which brings it nearer to its former habitat (Jennings). But the original physiological state can not be perfectly reestablished and made to persist in regular activity until the little animal by its own movements has succeeded in getting again into an environment identical with the former one.

Likewise the diminution of histogenetic substance in the blood which prevents the continuance of the metabolic state hitherto active and stable, will provoke the affective tendency of hunger and all the acts of seeking and absorbing nourishment proceeding therefrom; but the normal

metabolic state can not be completely reestablished until hunger is allayed; that is to say, until the acts carried on for the purpose of seeking and absorbing nourishment and the processes of digestion have endowed the blood with the same intrinsic quality, hence the same proportion of histologic substance, as formerly.

As with all mnemonic evocations in general, a small part of a certain former complex environmental state is sufficient, if not to "satisfy" the associated affective tendency, at least to "discharge" it. That is why the sensations in so far as they represent parts of environmental conditions, become in a very special manner the "dischargers" of affective tendencies. But in this respect there is an essential difference between the "non-distance receptors" and the "distance-receptors" which Sherrington rightly emphasizes, so that a very significant phylogenetic advance was made when the latter gradually developed from the former. For the non-distance receptors (senses with direct contact) usually permit the immediate or almost immediate satisfaction of the affective tendencies which they "discharge." Frequently the sensation discharging a certain affective tendency is identical with its satisfaction. On the other hand the "distance-receptors" usually produce that particular state in which an affective tendency is discharged and held in suspense, and which we are now ready to investigate.

"Between touch and assimilation," says Spencer, "there exists in the lowest creature an intimate connection. In many Rhizopods the tactual surface and the absorbing surface are coextensive. The ameba, a speck of jelly having no constant form, sends out in this or that direction prolongations of its substance. One of these meeting with and attaching itself to some relatively fixed object, becomes a temporary limb by which the body of the creature is drawn forward; but if this prolongation meets with some

relatively small portion of organic matter it slowly expands its extremity around this, slowly contracts, and slowly draws the nutritive morsel into the mass of the body, which collapses around it and presently dissolves it. That is to say, the same portion of tissue is at once arm, hand, mouth, and intestine—shows us the tactual and absorbent function united in one."[3]

Sherrington in his turn says: "Animal behavior shows clearly that in regard to these two groups of receptors the one subserves differention of reaction, i. e., swallowing or rejection, of material already found and acquired, e. g., within the mouth. The other, the distance-receptor, smell, initiates and subserves far-reaching complex reactions of the animal anticipatory to swallowing, namely, all that train of reaction which may be comprehensively termed the quest for food. The latter foreruns and leads up to the former. This precurrent relation of the reaction of the distance-receptor to the non-distance receptor" (as well as the 'conative feeling' which the distance-receptor induces) "are typical."[4]

Accordingly non-distance-receptors occasion no "suspended" affective tendencies, no "conative feeling," but instead they bring about the immediate satisfaction of affective tendencies at the moment they are discharged, or the immediate accomplishment of the acts contributing to their satisfaction ("final or consummatory reactions," as Sherrington expresses it). Distance-receptors, on the other hand, discharge the affective tendency involved and keep it active during the entire time of expectation and during the whole series of acts required of the animal before it can carry out the last consummatory act which is to satisfy this affective tendency. Therefore in general

[3] Herbert Spencer, *The Principles of Psychology*, 4th ed., Vol. I, p.307. London, Williams and Norgate.

[4] C. S. Sherrington, *The Integrative Action of the Nervous System*, page 326 f. London, Constable, 1906.

only the distance-receptors but not the non-distance-receptors can bring about a more or less lasting condition of unfulfilled desire: "If all motive impulses could be at once followed up desire would have no place."[5]

Now the question arises how can we explain the fact that the affective tendencies discharged or evoked by the distance-receptors, nevertheless remain "suspended"; in other words, how is it that although they have been evoked and persist in this state, yet for a long time they occasion no actual performance of any of those consummatory acts which to be sure would not now have any result but to which they nevertheless impel, as is shown by the incipient performance of these acts? The beast of prey, for instance, whose appetite is aroused from afar by the scent and sight of his victim coming towards him without presentiment of danger and is whetted constantly more and more, nevertheless does not bound at once toward the longed-for victim, but waits motionless and trembling with all the muscles tense, until the poor victim has come within springing distance. What then prevents the affective tendency so evoked from being at once completely discharged in the consummatory act of springing upon the prey and tearing it to pieces?

This can only be the counteraction of a conflicting tendency by which the first tendency is prevented from accomplishing its consummatory act. And the conflicting tendency in this case can be only the combined result of all consummatory acts which were actually performed in the past at the first awakening of the affective tendency, but every time without result. Accordingly we may make the assertion that it was the "deception" at each premature activation of the affective tendency called forth by the dis-

[5] A. Bain, *The Emotions and the Will*, 4th ed., p. 423. London, Longmans Green, 1899.

tance-receptor, which called into being the opposite tendency by which the other is now held in suspense.

A familiar instance is Möbius's experiment with the pike. By means of a pane of glass he divided a large glass bowl full of water into two parts. In one side he placed the pike and in the other he put tiny whitings which provide the pike's customary food. It now happened that whenever the pike dived after one of the small fishes he fell against the obstructing pane of glass. For a week he continued to make these vain attempts. Then he gave up entirely the pursuit of his unattainable prey and did not change his behavior even when the obstructing pane of glass had been taken away.

Now the constantly repeated deceptions which resulted when the affective tendency released by a distance-receptor produced immediately the performance of a consummatory act which was necessarily unsuccessful, must have a very similar effect on all animals provided with these senses. And so it has come to pass that the very discharge effected by the distance-receptors of any affective tendency and the premature beginning of the movement connected with it, now, thanks to the memory of former unsuccessful attempts, provoke the antagonistic tendency, like that which prevented the pike from falling upon its prey. And this conflict produces that state of an affective tendency "held in suspense" which constitutes the state of attention.

Accordingly we may say that phylogenetically attention originated with the distance-receptors, and that it consists in the conflict of two affective tendencies, the second of which is "discharged" by the first, prevents it for a time from complete activation and hence keeps it "in suspense."

The state of attention therefore does not consist of a single affective state but of the conflict of tendencies arising from the coexistence of two affective states. It is because this fact has been overlooked that it has not been pos-

sible heretofore to understand in what the specific nature of this state of attention really consists, and so to understand the real significance of the holding of an affective tendency "in suspense" which is characteristic of attention, nor to understand why all those movements which the first of the two affective tendencies would itself have provoked at once, are arrested "in the nascent state," whereas had this affectivity alone been active they would have proceeded directly to completion.

But aside from the case just considered of a premature performance of the consummatory act involved, the distance-receptors under many other circumstances arouse a second affectivity in conflict with the first which for some time prevents the complete activation of the former, as a consequence of the unexpected, unpleasant results which had some time previously been associated with it. However and whenever such an affective conflict occurs there at once arises also a corresponding state of attention; and *vice versa,* there is no state of attention without such a conflict of tendencies. For we need only consider carefully a few of the most significant cases, selected so as to be as different as possible from one another, in order at once to see in operation this conflict of tendencies in every state of attention.

"A young chick two days old, for example," says Lloyd Morgan, "had learned to pick out pieces of yolk from others of white of egg. I cut little bits of orange-peel of the same sizes as the pieces of yolk and one of these was soon seized but at once relinquished, the chick shaking its head. Seizing another he held it for a moment in the bill but then dropped it and scratched at the base of his beak. That was enough. He could not again be induced to seize a piece of orange-peel. The obnoxious material was now removed and pieces of yolk of egg substituted but they were left untouched, being probably taken for orange-peel. Sub-

sequently he looked at the yolk with hesitation, but presently pecked doubtfully, not seizing but merely touching. Then he pecked again, seized, and swallowed it."[6]

Accordingly we see here how the first act of attention of the newly hatched chicken arose from the conflict between its first tendency to seize the yolk of the egg and the conflicting tendency aroused by the memory of the unpleasant experience produced by picking up the orange-peel. The "effective guidance and control of consciousness," of which Lloyd Morgan speaks as one factor which influenced the instinctive pecking of the chicken, was thus only the arousing of a new affectivity, repugnance, that inhibited the first affectivity, hunger, which of itself impelled toward the completion of the instinctive act.[7]

A little girl is taken out walking by a servant. The child unexpectedly catches a glimpse of her mother on the other side of the street and wishes to run over to her at once. But the maid warns her with a cry, "Look out for the carriage!" and the little one stops. The carriage has hardly passed and she has almost taken a step ahead when another approaching vehicle forces her to give way again. The conflict of the two tendencies of hope and fear, kept alive in the child by the sight of her mother and the repeated passing of vehicles, is shown very clearly by the direction of her steps first forward and then backward. It is faithfully reflected in the expression of the small bright eyes which shine with anticipation and joy as soon as they are turned upon her mother and the child takes a step nearer to her, but at once look anxious and confused when they observe one of the heavy wagons of which there seems to be no end. Finally, however, the street-crossing is unobstructed. The state of fear and also the "state of attention," has entirely disappeared so that the

[6] Lloyd Morgan, *Habit and Instinct*, p. 40 f. New York, Arnold, 1896.

[7] Lloyd Morgan, *op. cit.*, pp. 129-131, 135, 139 f.

little girl may at last satisfy her wish and throw herself into her mother's arms.

The conflict of tendencies is likewise exhibited with great distinctness in certain typical states of attention where it is expressed in the exceedingly subtle choices between almost imperceptible modalities of a certain act.

A billiard player, for instance, who has already directed his cue at the ball, wishes first of all to make a successful stroke. He is ready to make the stroke but the extreme tension of the muscles in his arm causes him to fear that the stroke may turn out to be too strong, as it did shortly before. In consequence of this conflicting affectivity his muscles become somewhat lax. Nevertheless the weaker tension he now feels reawakens in him the memory of an earlier unsuccessful stroke when the movement of the ball had not been swift enough, and now he finds himself perplexed by the opposite fear lest the stroke may be too weak. By the swings of his arm, now longer and now shorter, which precede the stroke and bring the point of the cue nearer to the ball or farther from it, a spectator can discern the rapid alternation of conflicting affectivities which discharge each other and exaggerate or moderate each other in order finally to bring about the result of giving to the ball exactly the necessary force.

The same is true when a person who is writing attempts to remove with his finger a tiny hair from his steel pen. This rarely succeeds at the first attempt because the fear of soiling his finger-tips causes him to press them together before they are near enough to the point of the pen and the hair. The first failure gives rise to care lest the second attempt may also fail, and this opposite fear partly suppresses and moderates the fear of soiling the fingers, so that the wish to remove the hair by this time lends to the arm and fingers exactly the degree of muscular contraction

necessary to get hold of the extending end of the hair without touching the inky pen.

From this conflict of tendencies, inevitably occurring as soon as we attempt to perform an act "carefully," arises the well-known fact that attention, when directed to actions which by long practice have become mechanical, makes their execution less rapid and perfect than if they had taken place quite automatically.

"An automatic connection of contents or movements has nothing to gain from the intervention of attention,—nay suffers a very positive loss in accuracy and rapidity of realization, if the attention be directed upon it."[8]

Thus the recitation of a poem which has been learned so well by heart that it can be repeated mechanically becomes uncertain and hesitating when the speaker gives it his whole attention. And a person who writes his name with the greatest facility when he gives no thought to it is pretty sure to do it disconnectedly and without ease as soon as some one asks him for his autograph. For in this case every stroke of the pen needs a short preparation and requires a certain application of the will to begin and complete it, whereas the transference from one stroke to another becomes studied and awkward instead of easy and running as usual.[9]

Nevertheless there are individual cases, even where the attention is greatly aroused, in which the conflict of tendencies appears less distinct. For instance in Sardou's drama, "Tosca," we have the scene where Tosca's lover is tortured. It arouses the keenest sympathy and attention of all the spectators. Where is there any conflict of tendencies in this case? And yet we shall find it if we reflect a little. On the one hand there is the tendency, according

[8] O. Külpe, "The Problem of Attention," *Monist*, XIII, p. 61. Chicago, Oct. 1902.

[9] H. Maudsley, *The Physiology of Mind*, p. 520 f. London, Macmillan, 1876.—*The Pathology of Mind*, p. 143. London, Macmillan, 1895.

to the character of the spectator, either to fall upon the crafty Scarpia and slay him, or to throw oneself at his feet and with Tosca beg his mercy for her lover; or one might hasten to the aid of the unfortunate man and liberate him after driving away or killing the agents of the torturer. On the other hand the cultured man has acquired a tendency by education or custom to do nothing which conventionality does not permit, and not to make himself ridiculous by acts which would be the more ridiculous since every one knows that he is not beholding a reality but a mere invention. And that this is really the case is proved by the village theaters where the actor who plays the part of the tyrant is often hissed by the public, and sometimes even becomes the target of more or less harmless missiles thrown by the more unsophisticated spectators. The author once attended such a spectacle. Some conspirators were in hiding behind a curtain, waiting to kill the king, who by this time had won the favor of the public by his generosity and fearlessness. He had hardly appeared when a voice was heard to call out at the first movement of the curtain, "Look out, they are going to kill you!" The entire audience laughed uproariously, and the simple spectator was overcome with confusion. He will doubtless succeed another time in repressing his magnanimous outburst, thanks to the conflicting tendency not to make himself again the object of derision.

Attention which is aroused by novelty is likewise the result of a conflict of tendencies arising from the fact that just because the object is new, it has not yet been "affectively classified," and therefore arouses both hope and fear at the same time.

If the space at our disposal permitted, we could easily show that any "classification" whatever is based either directly or indirectly upon an affective tendency. The principle upon which it rests consists originally in the fact that

no sensation or perception of the distance-receptor has any value for the organism except as a symbol of a possible environmental state, near or remote, to be striven after or avoided. As long as this symbol has not been placed in either category, the conflicting affectivities of hope and fear oppose each other and hold each other in suspense. This opposition is seen distinctly, for instance, in a child who is undecided whether or not he should drink the tea offered him by his mother and which this time has an unusual color, because he is not sure whether it is a sweet or bitter draught; or in a beast of prey that sees a strange looking animal and is in doubt whether it is a dangerous enemy or perhaps a suitable quarry and therefore makes its muscles tense, ready at the same time for either attack or flight.

Curiosity is only one of the least forms of this conflict of tendencies or of this particular state of attention produced by novelty. "The craving for knowledge in its instinctive form is called curiosity. It exists in all degrees, from that of the animal which touches or smells an unknown object, to the all-examining, all-embracing scrutiny of a Goethe." "Curiosity consists of two questions expressed or implied: What is it? What use is it?....The dog brought face to face with an unknown object, looks at it, smells it, approaches, withdraws, ventures to touch it, returns, and begins again; he is pursuing this investigation after his own fashion; he is solving a double problem of nature and utility."[10]

On the other hand the "not new"—and this also may be any specific object when we see it for the first time—comprises everything we know how to classify in one of our various affective categories. It either brings about immediately the evocation and satisfaction of the affectivity con-

[10] Th. Ribot, *Psychologie des sentiments*, pp. 369, 371. Paris: Alcan, 1906. Second English edition, pp. 368, 370. London, Walter Scott, 1911.

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cerned, like the little waterfall in the mountain which awakens the desire to drink from it; or it evokes the affective tendency but holds it in suspense for fear lest its immediate complete satisfaction might involve some evil consequences as we have previously seen; or finally it may at that moment be altogether unable to evoke any tendency, like the sight or odor of a familiar dish when we have had enough. In this case the affective activity is reduced to a minimum, the state of attention entirely ceases, and we experience *monotony* or *tedium*. If this state of minimum affective activity is reduced to zero, we have the condition of *sleep*. "Sleep," as Bergson very truly says, "means to disinterest oneself (*se désintéresser*). We sleep in direct proportion to our disinterestedness."[11]

Finally there is only a very slight distinction between "curiosity" and the state of attention of the investigator. The investigator observes a certain object or a certain phenomenon in order to convince himself whether this object or this phenomenon really proves to possess certain properties whose presence has been asserted by others, or which he himself thought he noticed at the first glance, or which in his opinion should exist. The presence or absence of these properties is of exceedingly great value to the observer as is apparent from the fact that he applies himself with such great care to observe them, for they may for instance confirm certain preconceived theories or represent a highly important scientific discovery. Hence on the one hand he cherishes the ardent hope that the supposed properties would really be found to exist. On the other hand he is restrained from prematurely making known a discovery whose accuracy might later be contested by other inquirers to the great injury of his own scientific prestige. Just think for instance with what great

[11] H. Bergson, "Le rêve," *Bulletin de l'Institut Psychologique International*, p. 118. Paris, Alcan, May 1900.

attention—that is to say, with what great care lest he may have been a victim of an optical illusion—Schiaparelli must have carried on his ovservations before he decided to make known his discovery of the canals of Mars. Here too this hope and this care furnish the conflict of two affectivities without which here as elsewhere no actual state of attention would or could be present.

As we have by this time come to recognize the inmost nature of the affective conflict which, as appears from the few examples here adduced, is characteristic of every state of attention, so all other properties which always accompany this state prove at the same time to be so many simple and direct consequences of its nature.

Especially are we able to perceive at once the unconvincing character of Ribot's definition of attention as the state of "relative monoideism." We might if necessary call it a state of "monoaffectivity held in suspense," but as we have seen, it is still better to define it as a state of "double conflicting affectivity."[12]

Ribot's motor or peripheral theory proves to be equally erroneous: "Are the movements of the face, the body and the limbs, and the respiratory modifications that accompany attention, simply effects, outward marks as is usually supposed? Or are they, on the contrary, the necessary conditions, the consistent elements, the indispensable factors of attention? Without hesitation we accept the second thesis."[13]

On the other hand the so-called theories of "central origin" seem to be perfectly correct.[14] Attention is indeed a "central," psychological phenomenon; for the awakening of the primary or active affectivity and the counter-awak-

[12] See Th. Ribot, *Psychologie de l'attention*, pp. 6-8, 6th edition. Paris Alcan, 1902. English edition, p. 10.

[13] Ribot, *op. cit.*, p. 32. English edition, p. 25.

[14] See, e. g., J. Sully, "The Psycho-Physical Process in Attention," *Brain*, July 1890, especially pp. 155-157. London, Macmillan.—Vaschide and Meunier, *La Psychologie de l'attention*, pp. 196 f. Paris, Blond ,1910.

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ening of the secondary affectivity which holds the other in suspense, are phenomena of this nature. Attention therefore is first of all an essentially affective phenomenon and only indirectly and in a subordinate manner does it become a motor phenomenon by the fact that the awakening of any affectivity whatever always produces motor and peripheral phenomena which are therefore only accompanying or derived phenomena.

Ribot's error comes from the fact that he has not succeeded in correctly comprehending the nature of affective tendencies, for he sees very well that "attention always depends upon affective states," but he adds soon after: "How are we to represent to ourselves these tendencies? The only positive idea that we can get of them is to consider them as movements (or as inhibitions of movements), be they real or nascent."[15]

Accordingly for this inquirer the motor elements would by themselves constitute the entire essence of affective tendencies. But it is the affective tendencies which are the foundation of the motor elements, and the reverse is false.

As we have seen in our frequently cited treatise "On the Mnemonic Origin and Nature of Affective Tendencies," an affective tendency is only a gravitation, so to speak, toward that environment or those environmental relations which permit the reactivation of the mnemonic accumulation constituting this affective tendency. But of itself, it does not produce any preferential impulse toward one rather than toward another series of movements. For even if these movements were such as could eventually bring the organism back into the desired environmental conditions, yet in themselves they have nothing to do with the ultimate satisfaction of this affective tendency. It is only when one series of movements succeeds in bringing the organism back to the requisite environmental condi-

[15] Ribot, *Psychology of Attention*, pp. 166, 172. English edition, pp. 112, 116.

tions sooner or better than the others and only from this moment, that it becomes preferred to the others. Only from this moment will the awakening of the affective tendency give rise to definite motor elements.

But before this occurs, that is to say before the affective tendency has found preferable any one of the movements capable of leading to the desired end, the affective tendency towards that end will already exist. The very fact of this affective choice proves that in point of time the choosing factor precedes the element chosen, whence it follows that there can be an affective tendency even in the absence of any motor element. For instance a new and unusual indisposition which may attack us arouses the affective tendency to be freed from it, but this does not and cannot initiate any motion whatever.

Hence if affective tendencies and motor elements are two different things, and if the latter are based upon the former but not the reverse, then this is also true with regard to attention for which the motor elements are not an indispensable condition but merely quite secondary phenomena.

Since every conflict of affective tendencies is expressed in a conflict of the motor elements induced by them, so a clear explanation is afforded even with the "central origin" for the fact that "muscular tension," "motor innervation," "tonic contraction," and the "elevation of the entire psychic life," characterize every state of attention, as all have observed.[16]

Affective choice determines not only the particular movements of locomotion, of seizing, etc., which make for the desired object, but also the adjustment of the sense-organs, itself a musculo-motor phenomenon on which depends the more or less successful result of the movements,

[16] Maudsley, *The Physiology of Mind*, p. 313.—Ch. Féré, "Physiologie de l'attention," *Revue philosophique*, Oct. 1800, pp. 401, 404.—K. B. R. Aars, "Notes sur l'attention," *Année psychologique*, VIII, p. 216. Paris, Schleicher, 1902.

of whatever kind they are, and in which therefore both of the two conflicting affectivities cooperate. Now for instance when we are surprised by a sudden noise and direct our glance at once to the distant object from which it seems to come, the state of attention is alert during the whole interval preceding the moment in which the eyes have become adjusted to the new distance, which requires a certain length of time when the object is far away. Thus attention is awakened (here too in conformity with the theory of central origin) before and not after the adjustment of the organ concerned.[17]

Since on the other hand the peripheral sensory relations remain the same, the attention may be directed now to some and now to other sense-perceptions, just as when, confined within our room, we give more heed to certain noises in the street than to others which come from the same direction; for instance, to the hoof-beat of the horses belonging to an equipage that stops before our door, in order to determine by the sound which of our friends has come to call; or to the roll of the wheels in order to find out whether the friend who has come to take us out driving is riding in a closed or open carriage. Attention may even be directed to certain properties of a sense-impression, for instance to the strength or pitch of a note of music, or to certain other characteristics such as its *timbre*. No other examples could demonstrate better than these how entirely attention is independent of the adjustment of the sense, as well as in general of every other "peripheral factor."[18]

From this "central origin" of attention which has been so fully established, and from the inmost nature of the opposition between two mutually conflicting affectivities as above discussed, a conclusion of the utmost importance

[17] See W. B. Pillsbury, *Attention*, p. 13. London, Swan Sonnenschein, 1908.

[18] O. Külpe, *loc. cit.*, p. 50.

may be drawn, namely that the object of attention is observed simultaneously from two quite distinct points of view. Thus a large number of properties and characteristics, of advantages and disadvantages are perceived, observed, recalled and emphasized, which would by no means be the case if only a single affectivity were operative.

Wundt's well-known metaphorical definition of the "apperception" produced by attention as consisting in the transition of the image "from the internal visual field to the internal visual point of consciousness," accordingly, might better be replaced by that of an internal double reflector illuminating the object or the image from several sides at the same time.[19]

That is why attention prevents the mnemonic addition of sensation-evocations, which the affectivity adds to the rough elementary sensation at the moment it is aroused, from distorting the perception produced by this mnemonic contribution into an illusion or hallucination, which on the contrary is always the case when the affectivity thus aroused remains alone.

Sudden and intense fear, for instance, makes any state of attention quite impossible and may give rise—as in the classical case of the wanderer walking at night through a dense forest—to those characteristic hallucinations cited and described in all text-books of psychology and psychopathology. On the other hand that man is "cold-blooded" who does not flee at the sudden rustling of leaves which arouses in him at the first moment the vision of some hidden robber or dangerous beast behind the trees, but who, restrained by his repugnance to so cowardly an action, looks around "with attention" to see whether there really is a living creature there, and what sort of a one it is, or whether indeed it was not the wind that made the noise.

[19] W. Wundt, *Grundzüge der physiologischen Psychologie*, 5th ed., Vol. III, p. 333. Leipsic, Engelmann, 1903.—Ostwald, *Vorlesungen über Naturphilosophie*, 3d ed., pp. 400, 403. Leipsic, Veit, 1905.

Likewise in a state of passion any attention to all that is connected with this passion becomes impossible and the passionate man is therefore exposed to all the auto-suggestions and hallucinations of an Othello because of the very singleness of the control by the hypertrophic affective tendency characteristic of this state. In monomaniacs also as well as in those suffering from a chronic persecution-mania and similar psychical diseases, the thing lacking is the counter-affectivity which would tend to make them fear that they were making a mistake. They are mono-affective in the proper sense of the word, therefore incapable likewise of a real and proper state of attention.

The absence of any counter-affectivity produces in all these cases a total absence of "opposing inhibitors," as Taine would say, which could inhibit the auto-suggestions and hallucinations produced by the one existing affectivity, and permit the latter to reign unhindered and exclusively. On the other hand, great attention always protects from suggestion practised by others just because the opposite affectivity, the fear of being deceived, becomes very strong, as is proved for instance by Binet's experiments on the susceptibility of school children to suggestion.[20]

* * *

Now as we pass to the relations existing between attention and consciousness we must first briefly mention our theory with regard to the conditions which determine the consciousness and those which determine the unconsciousness of the different psychic states.[21]

In the above mentioned treatise we have come to the conclusion that a given psychic state is neither conscious nor unconscious in itself, but that it seems to possess either

[20] H. Taine, *De l'intelligence*, 8th ed., Vol. I, pp. 95 ff. Paris, Hachette, 1897.—A. Binet, *La suggestibilité*, pp. 166, 177 f., 186, 191, 196, 200 etc. Paris, Schleicher, 1900.

[21] E. Rignano, "Qu'est-ce que la conscience?" *Scientia*, 1907, Vol. II, No. IV, 4.

one or the other of these properties only when, having been previously present, it is now referred to another psychic state at present existing. And the necessary and sufficient condition permitting a complex past psychic state to present itself again as "conscious" in relation to a complex present psychic state is that the affective portion of the mnemonic evocation of the former correspond at least in part with the coexisting affective portion of the latter and therefore coalesce with it.

Since, as we have seen in our frequently cited treatise, the possession of a "diffuse seat" is characteristic of affective tendencies—which in this respect are so different from sensations and their images whose seat is localized at a single point or center and which therefore may exist and be active simultaneously in great numbers in one and the same brain—it is difficult even for only two affective tendencies to have their seats in localities which shall not coincide more or less, so that when these tendencies strive to be operative at the same time, they either conflict with each other, or hold each other in suspense, or partially coalesce.

If the discharge of one does not depend on the discharge of the other, and if the respective nervous activities in the part of their seats common to both differ specifically from each other, then the activation of one tendency will of itself imply the exclusion of the other and *vice versa*. If the discharge of the one is caused by the discharge of the other and the two tendencies are antagonistic, we will then have the state in which the primary affective tendency is held in suspense by the secondary; which condition, as we have seen above, is characteristic of the state of attention. If on the other hand the respective nervous activities in that portion of their seats common to both are specifically similar, then their blending together will make the complex psychic state to which one of the tendencies belongs "con-

scious" with reference to the psychic state to which the other belongs.

Finally a fourth case will occur but much more rarely for reasons given above, in which the two affective tendencies have no part of their seats in common, and accordingly both can be present and operative at the same time without hindering each other or bearing any relation whatever to one another. This case comprises all the phenomena of the so-called double personality. These phenomena nevertheless are by no means always of a pathological character, like the typical ones studied especially by Janet, but they may appear also in normal persons in so-called instances of absent-mindedness. Such was the case, for instance, when we were climbing down into the valley from Ca' di Janzo by a very steep mule path. Leaping from one stone to another constantly demanded our whole attention in order to measure exactly the distance of the leap and lest a foot should slip or dislodge a stone. Yet nevertheless the descent sometimes proceeded "unconsciously" with reference to some other very different affectivity which produced at the same time quite another train of thought.[22]

In the first case the exclusion of all other tendencies with independent discharge as soon as one of them becomes active—an exclusion which persists throughout the whole time during which the first of the two affective tendencies of the state of attention remains "held in suspense"—forms the so-called "unity of consciousness."

In other words, the impossibility for more than one primary affective tendency to be active at any one time results in the impossibility of giving heed to more than one object at one time: "A plurality of stimulations of the nerves may co-exist, but they affect the consciousness only

[22] P. Janet, *L'automatisme psychologique*, pp. 263 ff. Paris, Alcan, 1907... Taine, *De l'intelligence*, pp. 16 ff.—Rignano, *Qu'est-ce que la conscience?* pp. 11-13.

by turns, or one at a time. The reason is that the bodily organs are collectively engaged with each distinct conscious state, and they cannot be doing two things at the same instant."[23]

Consequently attention ordinarily is never divided or dispersed. If it is greatly roused it will continue to be directed toward any given objects for a while and hence can not be directed to any others during this entire period. If it is less aroused it passes from one object to another in quick succession and accordingly seems to be divided among many objects at the same time; but in reality even in this case it is directed at each moment to one object only, that is, to the one which corresponds to the momentary affective tendency. Accordingly the speaker who passes judgment upon his own speech, the actor who has command over himself, the chess player who plays several games at one time, Julius Cæsar who dictated several letters at once, do not prove the simultaneous presence of several states of attention, but rather their rapid succession and the alternating predominance of first one and then another.[24]

For this reason the attention directed by self-contemplation upon any affective state brings about the end and disappearance of that state. It is impossible to direct one's attention upon an affectivity. If the attempt is made that particular mood ceases at once, and we are turned aside by a compelling sensation or idea which we have not the slightest desire to observe.[25] For the attention which is directed upon an affectivity within ourselves is a newly originated affectivity, namely the one that impels us to

[23] Bain, *The Emotions and the Will*, p. 5.

[24] E. Meumann, *Intelligenz und Wille*, pp. 22 ff. Leipsic, Quelle & Meyer, 1908.

[25] E. B. Titchener, *The Psychology of Feeling and Attention*, p. 69. New York, Macmillan, 1908.

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this observation and investigation, and therefore it displaces the other we wished to observe.

Since the primary affective tendency of the state of attention excludes every other affectivity independently evoked and in this way protects the unity of our consciousness, it makes it possible at the same time for every past state of attention involved to appear conscious to us if we now think back to it and to the object which at that time constituted the end desired. For this memory will now be recalled to the same object by a more or less similar affective tendency which therefore will partially blend with the recollection of the former.

Every state of attention accordingly contains all elements within itself in order later to seem to us to be conscious; but not all past psychic states which now appear conscious were states of attention, as Kohn maintains to whom the state of attention and the conscious state are the same thing. For an affectivity which becomes at once completely active and therefore does not give rise to any state of attention — like a hurried flight caused by sudden terror—is nevertheless able to make the complex psychic state involved appear a conscious one.[26] In other words, the state of attention is a sufficient but not a necessary condition of consciousness. The only condition which is at the same time necessary and sufficient is the presence of some affective tendency, no matter whether it be in the state of suspense or of full activation.

The acts which have become automatic, for instance those which originated through affective choice as conscious movements, and which later by means of attention were perfected under the affective conflict of the tendencies to perform the act but at the same time to avoid one by one its many imperfections, are finally consummated after

[26] See H. E. Kohn, *Zur Theorie der Aufmerksamkeit*, pp. 19, 27. Halle, Niemeyer, 1895.

frequent repetition—according to the mnemonic law that the part gradually becomes independent of the whole—without requiring any "impulsion" or any kind of affective aid whatever, either primarily in the execution or secondarily by way of improvement. For this reason we are accustomed to say that rendering acts automatic liberates the attention so that it may be directed to other objects.[27]

And just because acts which have become automatic do not require attention on our part and take place without the assistance of any affective element, they always seem to us to be unconscious. Consciousness, as Maudsley says, directs the process of adaptation, the efforts to become expert in adjusting the various means to their proper ends and the successive stages of organization; it disappears as soon as the skill has been thoroughly attained.[28]

"Habit," says James, "diminishes the conscious attention with which our acts are performed. One may state this abstractly thus: If an act require for its execution a chain of successive nervous events, then in the first performances of the action the conscious will must choose each of these events from a number of wrong alternatives that tend to present themselves; for consciousness is always and chiefly a selective agency. But habit soon brings it about that each event calls up its own appropriate successor without any alternative offering itself and without any reference to the conscious will, until at last the whole chain rattles itself off as soon as the first event occurs, just as if this and the rest of the chain were fused into a continuous stream."[29]

Just as an act that has become automatic represents a nervous activity which in the absence of any accompanying

[27] Meumann, *Intelligenz und Wille*, p. 23.

[28] Maudsley, *The Pathology of Mind*, p. 9.

[29] Wm. James, *The Principles of Psychology*, Vol. I, pp. 114, 139. London, Macmillan, 1901. The same, briefer course, p. 139. New York, Holt, 1893.

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affective tendency remains unconscious, so will every stimulation of our senses remain unconscious when it reaches its sensory seat if it can not arouse any affectivity in us. On the other hand every stimulation of our senses which succeeds in discharging any one of the many affective tendencies potentially present in the brain, will afterwards appear conscious to us; and this may also be expressed by saying that the "stimulation has succeeded in taking possession of the sensorium."[30] Whence it follows that if all objective and sensitive peripheral relations remain the same, it will depend on whether our attention is or is not directed upon something else and on the degree of strength and of opposition of the primary affectivity involved—for thence is derived the power to exclude every other affective tendency which differs from it—whether certain stimuli remain quite unobserved or whether they will appear to us as conscious sensations.[31]

Says James: "A million things in the outside world are present to my senses but do not enter my consciousness. Why? Because they do not interest me. Only that which arouses my attention makes up my experience. Only the objects to which I give heed constitute my understanding. Without selective interest experience is a veritable chaos. Interest first gives color and tone to the image, light and shadow, background and foreground, in a word a distinct perspective."[32]

The primary affectivity of a state of averted attention may be so strong that it can prevent even the most intense irritations, which at other times would seem altogether painful and arouse within us the most strenuous effort to remove them, from reaching our consciousness. Classical,

[30] G. E. Müller, *Zur Theorie der sinnlichen Aufmerksamkeit*, pp. 77. Leipsic, Edelmann.

[31] Müller, *op. cit.*, p. 1.—Külpe, *op. cit.*, p. 40 f.—Ostwald, *Vorlesungen über Naturphilosophie*, pp. 400 ff.

[32] James, *op. cit.*, Vol. I, p. 402.

for instance, is the case of the Christian martyr whose entranced attention was to such a degree absorbed by the beatific visions presented to his eyes, that it prevented him from feeling the pain of the horrible tortures to which his body was subjected. No less significant is the case of Robert Hall, some of whose "most eloquent discourses were poured forth whilst he was suffering under a bodily disorder which caused him to roll in agony on the floor when he descended from the pulpit; yet he was entirely unconscious of the irritation of his nerves by the calculus which shot forth its jagged points through the whole substance of his kidney, so long as his soul continued to be 'possessed' by the great subjects upon which a powerful effort of his will originally fixed it."[33]

However, a large number of facts go to prove that those very irritations which do not discharge any affectivity or are not capable of arousing our attention and therefore remain unconscious, nevertheless likewise succeed in reaching their sensory seats. "The fact that we sometimes become conscious of many sensuous impressions, such as for instance the stroke of a bell, after the stimulus has made itself felt in our sense-organ, tends to show that the excitation reaches its destination rightly enough, but that the sensory center happens at the moment to be in a state not suited for the reception of the afferent stimulus."[34]

The conflict also between the different states of attention which the varied stimuli from the outside world would strive to arouse—owing to the fact that only one single primary affective tendency can ever be operative at any one moment—indicates that, whatever the relation of the stimulations to consciousness may be, they always reach their habitual psychical center; for otherwise they could not all tend to discharge their respective affectivities.

[33] W. P. Carpenter, *Principles of Mental Physiology*, 7th ed., p. 138. London, Kegan Paul, 1896.

[34] Müller, *Zur Theorie der sinnl. Aufm.*, p. 105.

"When one of the various stimuli succeeds in the struggle to obtain possession of consciousness we say that we are attentive to it according to the intensity of the corresponding process of consciousness." "But we can not maintain that excitations which do not enter our consciousness because of averted attention do not enter at all into the organ of consciousness, the cortex of the brain."[35]

It often happens in my own case, for instance, that I am reading a newspaper while the other members of the family are chatting together in the same room or perhaps while one of them reads aloud from a book or a different paper. Sometimes I do not succeed in limiting my attention to what I myself am reading because my interest is aroused by what I hear read aloud. In other cases, however, I succeed very well, and then I no longer hear the words of those in the room. Nevertheless one word pronounced by the reader in exactly the same tone as all the other words—for he is reading right along in the same monotonous voice—suddenly draws me completely away from what I am reading and turns my attention to what he is reading aloud. Thus my attention vibrates constantly back and forth between what I am reading and what I am hearing read. The fact of this conflict between the two states of attention accordingly proves most positively, I repeat, that the irritations produced by the spoken words of another reach their sensory center, their sensory basis, in me even in moments when I am not aware of them; otherwise none of them would be able to rivet my interest or attention.

The same is obviously true for all so-called states of absentmindedness which at bottom, as we have already seen, are only the first physiological indications of that double state of one's own personality which hitherto has

[35] Kohn, *Zur Theorie der Aufm.*, p. 19; and Sigmund Exner, *Entwurf zu einer physiologischen Erklärung der psychischen Erscheinungen.* Part I, p. 72. Vienna and Leipsic, Deuticke, 1894.

been investigated almost exclusively in its pathological forms. As an example of this we mentioned in our essay on consciousness the locking of a drawer while attention was directed elsewhere. This showed that all stimulations of sight proceeding from the key-hole and the key placed in it reached their goal although they remained entirely unconscious. Every one has the experience of walking absentmindedly through the streets and yet without running into people, vehicles, or any other obstructing objects on the way. Our previously mentioned "unconscious" descent from the Ca' di Janzo proves how perfectly in every respect the perception of all the difficulties of the way must have been—the stones, their form, their position, their state of equilibrium—if I were to succeed in leaping from one stone to another without falling or knocking down a stone.

The primary affective tendency which constitutes that state of attention which is directed on a definite object, by no means excludes the intrusion of sensations which at the time have no interest; or, in other words, it does not prevent excitations of a sensory character from reaching their goal, their normal destination, even when we are unconscious of them; but they only oppose the affective tendency which would endeavor to arouse these sensations.

"The entrance of a stimulus into consciousness"—as it is expressed by Kohn and others—does not rest upon the possible intrusion of the stimulus at any particular part of the brain or sensorium whose specific function would be that of consciousness. No more does it depend upon a single "center of perception" as Wundt assumes. But it consists only in the fact that this stimulus evokes some affective tendency relating to the object which it represents. When this evocation takes place the stimulus reaches consciousness; if it does not take place, perhaps because at this moment another affective tendency referring to other

sensations is operative, then, although the stimulus may penetrate physiologically to the same point as usual, it cannot reach consciousness and hence remains unobserved and unconscious. The persistence of the mnemonic accumulations of those sensations which remain outside of consciousness and the possibility of evoking them again in the future are at a great disadvantage from the circumstance that they are not able to excite any affective state peculiar to themselves with which they could be connected or associated.

Having thus elucidated the inmost nature of the affective conflict peculiar to attention in its main points, and having seen wherein consists that unity of consciousness which so many inquirers declare to be one of its most essential fundamental properties, space does not now permit us to pass on to the study of the effects arising from this inmost nature and fundamental property of attention upon sensations and ideas, as in general for the whole process of intelligence.

E. RIGNANO.

MILAN, ITALY.

CHANCE.[1]

"HOW dare we speak of the laws of chance? Is not chance the antithesis of all law?" So says Bertrand at the beginning of his *Calcul des probabilités*. Probability is opposed to certitude; so it is what we do not know and consequently it seems what we could not calculate. Here is at least apparently a contradiction, and about it much has already been written.

And first, what is chance? The ancients distinguished between phenomena seemingly obeying harmonious laws, established once for all, and those which they attributed to chance; these were the ones unpredictable because rebellious to all law. In each domain the precise laws did not decide everything, they only drew limits between which chance might act. In this conception the word chance had a precise and objective meaning: what was chance for one was also chance for another and even for the gods.

But this conception is not ours to-day. We have become absolute determinists, and even those who want to reserve the rights of human free will let determinism reign undividedly in the inorganic world at least. Every phenomenon, however minute, has a cause; and a mind infinitely powerful, infinitely well-informed about the laws of nature, could have foreseen it from the beginning of the centuries. If such a mind existed, we could not play with it at any game of chance, we should always lose.

In fact for it the word chance would not have any mean-

[1] Translated by G. B. Halsted.

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ing, or rather there would be no chance. It is because of our weakness and our ignorance that the word has a meaning for us. And, even without going beyond our feeble humanity, what is chance for the ignorant, is not chance for the scientist. Chance is only the measure of our ignorance. Fortuitous phenomena are, by definition, those whose laws we do not know.

But is this definition altogether satisfactory? When the first Chaldean shepherds followed with their eyes the movements of the stars, they knew not as yet the laws of astronomy; would they have dreamed of saying that the stars move at random? If a modern physicist studies a new phenomenon, and if he discovers its law Tuesday, would he have said Monday that this phenomenon was fortuitous? Moreover, do we not often invoke what Bertrand calls the laws of chance, to predict a phenomenon? For example in the kinetic theory of gases we obtain the known laws of Mariotte and of Gay-Lussac by means of the hypothesis that the velocities of the molecules of gas vary irregularly, that is to say at random. All physicists will agree that the observable laws would be much less simple if the velocities were ruled by any simple elementary law whatsoever, if the molecules were, as we say, *organized,* if they were subject to some discipline. It is due to chance, that is to say to our ignorance, that we can draw our conclusions; and then if the word chance is simply synonymous with ignorance what does that mean? Must we therefore translate it as follows?

"You ask me to predict for you the phenomena about to happen. If, unluckily, I knew the laws of these phenomena I could make the prediction only by inextricable calculations and would have to renounce attempting to answer you; but as I may chance not to know, I will answer you at once. And what is most surprising, my answer will be right."

So it must well be that chance is something other than the name we give our ignorance, that among phenomena whose causes are unknown to us we must distinguish fortuitous phenomena about which the calculus of probabilities will provisionally give information, from those which are not fortuitous and of which we can say nothing so long as we shall not have determined the laws governing them. For the fortuitous phenomena themselves, it is clear that the information given us by the calculus of probabilities will not cease to be true upon the day when these phenomena shall be better known.

The director of a life insurance company does not know when each of the insured will die, but he relies upon the calculus of probabilities and on the law of great numbers and he is not deceived since he distributes dividends to his stockholders. These dividends would not vanish if a very penetrating and very indiscrete physician should, after the policies were signed, reveal to the director the life chances of the insured. This doctor would dissipate the ignorance of the director, but he would have no influence on the dividends which evidently are not an outcome of this ignorance.

* * *

To find a better definition of chance we must examine some of the facts which we agree to regard as fortuitous, and to which the calculus of probabilities seems to apply; we then shall investigate what are their common characteristics.

The first example we select is that of unstable equilibrium; if a cone rests upon its apex, we know well that it will fall, but we do not know toward what side; it seems to us chance alone will decide. If the cone were perfectly symmetric, if its axis were perfectly vertical, if it were acted upon by no force other than gravity, it would not fall at all. But the least defect in symmetry will make it lean slightly toward one side or the other, and if it leans,

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however little, it will fall altogether toward that side. Even if the symmetry were perfect, a very slight tremor, a breath of air could make it incline some seconds of arc; this will be enough to determine its fall and even the sense of its fall which will be that of the initial inclination.

A very slight cause, which escapes us, determines a considerable effect which we cannot help seeing, and then we say this effect is due to chance. If we could know exactly the laws of nature and the situation of the universe at the initial instant, we should be able to predict exactly the situation of this same universe at a subsequent instant. But even then when the natural laws should have no further secret for us, we could know the initial situation only *approximately*. If that permits us to foresee the subsequent situation *with the same degree of approximation*, this is all we require, we say the phenomenon has been predicted, that it is ruled by laws; but it is not always so. It may happen that slight differences in the initial conditions produce very great differences in the final phenomena; a slight error in the former would make an enormous error in the latter. Prediction becomes impossible and we have the fortuitous phenomenon.

Our second example will be very analogous to the first and we shall take it from meteorology. Why have the meteorologists such difficulty in predicting the weather with any certainty? Why do the rains, the tempests themselves seem to us to come by chance, so that many persons find it quite natural to pray for rain or shine, when they would think it ridiculous to pray for an eclipse? We see that great perturbations generally happen in regions where the atmosphere is in unstable equilibrium. The meteorologists are aware that this equilibrium is unstable, that a cyclone is arising somewhere; but where they cannot tell; one-tenth of a degree more or less at any point, and the cyclone bursts here and not there, and spreads its ravages

over countries it would have spared. This we could have foreseen if we had known that tenth of a degree, but the observations were neither sufficiently close nor sufficiently precise, and for this reason all seems due to the agency of chance. Here again we find the same contrast between a very slight cause, unappreciable to the observer, and important effects, which are sometimes tremendous disasters.

Let us pass to another example, the distribution of the minor planets on the zodiac. Their initial longitudes can have been any longitudes whatever; but their mean motions were different and they have revolved for so long a time that we may say they are now distributed *at random* along the zodiac. Very slight initial differences between their distances from the sun, or, what comes to the same thing, between their mean motions, have ended by giving enormous differences between their present longitudes. An excess of the thousandth of a second in the daily mean motion will give in fact a second in three years, a degree in ten thousand years, an entire circumference in three or four million years, and what is that to the time which has passed since the minor planets have detached themselves from the nebula of Laplace? Again therefore we see a slight cause and a great effect; or better, slight differences in the cause and great differences in the effect.

The game of roulette does not take us as far as might seem from the preceding example. Assume a needle to be turned on a pivot over a dial divided into a hundred sectors alternately red and black. If it stops on a red sector I win, if not, I lose. Evidently all depends upon the initial impulse I give the needle. The needle will make, suppose, ten or twenty turns, but it will stop sooner or not so soon according as I shall have pushed it more or less strongly. It suffices that the impulse vary only by a thousandth or a two thousandth to make the needle stop over a black sector or over the following red one. These are differences

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the muscular sense cannot distinguish and which elude even the most delicate instruments. So it is impossible for me to foresee what the needle I have started will do, and this is why my heart throbs and I hope everything from luck. The difference in the cause is imperceptible, and the difference in the effect is for me of the highest importance, since it means my whole stake.

* * *

Permit me, in this connection, a thought somewhat foreign to my subject. Some years ago a philosopher said that the future is determined by the past, but not the past by the future; or, in other words, from knowledge of the present we could deduce the future, but not the past; because, said he, a cause can have only one effect, while the same effect might be produced by several different causes. It is clear no scientist can subscribe to this conclusion. The laws of nature bind the antecedent to the consequent in such a way that the antecedent is as well determined by the consequent as the consequent by the antecedent. But whence came the error of this philosopher? We know that in virtue of Carnot's principle physical phenomena are irreversible and the world tends toward uniformity. When two bodies of different temperature come in contact, the warmer gives up heat to the colder; so we may foresee that the temperature will equalize. But once equal, if asked about the anterior state, what can we answer? We might say that one was warm and the other cold, but not be able to divine which formerly was the warmer.

And yet in reality the temperatures will never reach perfect equality. The differences of temperature only tend asymptotically toward zero. There comes a moment when our thermometers are powerless to make it known. But if we had thermometers a thousand times, a hundred thousand times as sensitive, we should recognize that there still is a slight difference, and that one of the bodies remains

a little warmer than the other, and so we could say this it is which formerly was much the warmer.

So then there are, contrary to what we found in the former examples, great differences in cause and slight differences in effect. Flammarion once imagined an observer going away from the earth with a velocity greater than that of light; for him time would have changed sign. History would be turned about, and Waterloo would precede Austerlitz. Well, for this observer, effects and causes would be inverted; unstable equilibrium would no longer be the exception. Because of the universal irreversibility all would seem to him to come out of a sort of chaos in unstable equilibrium. All nature would appear to him delivered over to chance.

* * *

Now for other examples where we shall see somewhat different characteristics. Take first the kinetic theory of gases. How should we picture a receptacle filled with gas? Innumerable molecules, moving at high speeds, flash through this receptacle in every direction. At every instant they strike against its walls or each other, and these collisions happen under the most diverse conditions. What above all impresses us here, is not the littleness of the causes, but their complexity, and yet the former element is still found here and plays an important rôle. If a molecule deviated right or left from its trajectory, by a very small quantity, comparable to the radius of action of the gaseous molecules, it would avoid a collision or sustain it under different conditions, and that would vary the direction of its velocity after the impact, perhaps by ninety degrees or by a hundred and eighty degrees.

And this is not all; we have just seen that it is necessary to deflect the molecule before the clash by only an infinitesimal, to produce its deviation after the collision by a finite quantity. If then the molecule undergoes two suc-

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cessive shocks, it will suffice to deflect it before the first by an infinitesimal of the second order, for it to deviate after the first encounter by an infinitesimal of the first order, and after the second hit, by a finite quantity. And the molecule will not undergo merely two shocks; it will undergo a very great number per second. So that if the first shock has multiplied the deviation by a very large number A, after n shocks it will be multiplied by A^n. It will therefore become very great not merely because A is large, that is to say because little causes produce big effects, but because the exponent n is large, that is to say because the shocks are very numerous and the causes very complex.

Take a second example. Why do the drops of rain in a shower seem to be distributed at random? This is again because of the complexity of the causes which determine their formation. Ions are distributed in the atmosphere. For a long while they have been subjected to air-currents constantly changing, they have been caught in very small whirlwinds, so that their final distribution has no longer any relation to their initial distribution. Suddenly the temperature falls, vapor condenses, and each of these ions becomes the center of a drop of rain. To know what will be the distribution of these drops and how many will fall on each paving-stone, it would not be sufficient to know the initial situation of the ions, it would be necessary to compute the effect of a thousand little capricious air-currents.

And again it is the same if we put grains of powder in suspension in water. The vase is ploughed by the currents whose law we know not, we only know it is very complicated. At the end of a certain time the grains will be distributed at random, that is to say uniformly, in the vase; and this is due precisely to the complexity of these currents. If they obeyed some simple law, if for example the vase revolved and the currents circulated around the axis of the vase, describing circles, it would no longer be the same,

since each grain would retain its initial altitude and its initial distance from the axis.

We should reach the same result in considering the mixing of two liquids or of two fine-grained powders. And to take a grosser example, this is also what happens when we shuffle playing-cards. At each stroke, the cards undergo a permutation (analogous to that studied in the theory of substitutions). What will happen? The probability of a particular permutation (for example that bringing to the nth place the card occupying the $\phi(n)$th place before the permutation) depends upon the player's habits. But if this player shuffles the cards long enough, there will be a great number of successive permutations, and the resulting final order will no longer be governed by aught but chance; I mean to say that all possible orders will be equally probable. It is to the great number of successive permutations, that is to say to the complexity of the phenomenon, that this result is due.

A final word about the theory of errors. Here it is that the causes are complex and multiple. To how many snares is not the observer exposed, even with the best instrument! He should apply himself to finding out the largest and avoiding them. These are the ones giving birth to systematic errors. But when he has eliminated those, admitting that he succeeds, there remain many small ones which, their effects accumulating, may become dangerous. Thence come the accidental errors; and we attribute them to chance because their causes are too complicated and too numerous. Here again we have only little causes each of which might produce only a slight effect; it is by their union and their number that their effects became formidable.

* * *

We may take still a third point of view, less important than the first two and upon which I shall lay less stress.

When we seek to foresee an event and examine its antecedents, we strive to search into the anterior situation. This could not be done for all parts of the universe and we are content to know what is passing in the neighborhood of the point where the event should occur, or what would appear to have some relation to it. An examination cannot be complete and we must know how to choose. But it may happen that we have passed by circumstances which at first sight seemed completely foreign to the foreseen happening, to which one would never have dreamed of attributing any influence and which nevertheless, contrary to all anticipation, come to play an important rôle.

A man passes in the street going to his business; some one knowing the business could have told why he started at such a time and went by such a street. On the roof works a tiler. The contractor employing him could in a certain measure foresee what he would do. But the passer-by scarcely thinks of the tiler, nor the tiler of him; they seem to belong to two worlds completely foreign to one another. And yet the tiler drops a tile which kills the man, and we do not hesitate to say this is chance.

Our weakness forbids our considering the entire universe and makes us cut it up into slices. We try to do this as little artificially as possible. And yet it happens from time to time that two of these slices react upon one another. The effects of this mutual action then seem to us to be due to chance.

Is this a third way of conceiving chance? Not always; in fact most often we are carried back to the first or the second. Whenever two worlds usually foreign to one another, come thus to react upon each other, the laws of this reaction must be very complex. On the other hand a very slight change in the initial conditions of these two worlds would have been sufficient for the reaction not to have

happened. How little was needed for the man to pass a second later or the tiler to drop his tile a second sooner.

* * *

All we have said still does not explain why chance obeys laws. Does the fact that the causes are slight or complex suffice for our foreseeing, if not their effects *in each case*, at least what their effects will be, *on the average*? To answer this question we had better take up again some of the examples already cited.

I shall begin with that of the roulette. I have said that the point where the needle will stop depends upon the initial push given it. What is the probability of this push having this or that value? I know nothing about it, but it is difficult for me not to suppose that this probability is represented by a continuous analytic function. The probability that the push is comprised between a and $a+\epsilon$ will then be sensibly equal to the probability of its being comprised between $a+\epsilon$ and $a+2\epsilon$, *provided* ϵ *be very small.* This is a property common to all analytic functions. Minute variations of the function are proportional to minute variations of the variable.

But we have assumed that an exceedingly slight variation of the push suffices to change the color of the sector over which the needle finally stops. From a to $a+\epsilon$ it is red, from $a+\epsilon$ to $a+2\epsilon$ it is black; the probability of each red sector is therefore the same as of the following black, and consequently the total probability of red equals the total probability of black.

The datum of the question is the analytic function representing the probability of a particular initial push. But the theorem remains true whatever be this datum, since it depends upon a property common to all analytic functions. From this it follows finally that we no longer need the datum.

What we have just said for the case of the roulette

applies also to the example of the minor planets. The zodiac may be regarded as an immense roulette on which have been tossed many little balls with different initial impulses varying according to some law. Their present distribution is uniform and independent of this law, for the same reason as in the preceding case. Thus we see why phenomena obey the laws of chance when slight differences in the causes suffice to bring on great differences in the effects. The probabilities of these slight differences may then be regarded as proportional to these differences themselves, just because these differences are minute, and the infinitesimal increments of a continuous function are proportional to those of the variable.

Take an entirely different example, where intervenes especially the complexity of the causes. Suppose a player shuffles a pack of cards. At each shuffle he changes the order of the cards, and he may change them in many ways. To simplify the exposition, consider only three cards. The cards which before the shuffle occupied respectively the places 123, may after the shuffle occupy the places

$$123,\ 231,\ 312,\ 321,\ 132,\ 213.$$

Each of these six hypotheses is possible and they have respectively for probabilities:

$$p_1,\ p_2,\ p_3,\ p_4,\ p_5,\ p_6.$$

The sum of these six numbers equals 1; but this is all we know of them; these six probabilities depend naturally upon the habits of the player which we do not know.

At the second shuffle and the following, this will recommence, and under the same conditions; I mean that p_4 for example represents always the probability that the three cards which occupied after the nth shufflle and before the $n+1$th the places 123, occupy the places 321 after the $n+1$th shuffle. And this remains true whatever be the number n, since the habits of the player and his way of shuffling remain the same.

But if the number of shuffles is very great, the cards which before the first shuffle occupied the places 123 may, after the last shuffle, occupy the places

123, 231, 312, 321, 132, 213

and the probability of these six hypotheses will be sensibly the same and equal to 1/6; and this will be true whatever be the numbers $p_1 \ldots . p_6$ which we do not know. The great number of shuffles, that is to say the complexity of the causes, has produced uniformity.

This would apply without change if there were more than three cards, but even with three cards the demonstration would be complicated; let it suffice to give it for only two cards. Then we have only two possibilities 12, 21 with the probabilities p_1 and $p_2 = 1 - p_1$.

Suppose n shuffles and suppose I win one franc if the cards are finally in the initial order and lose one if they are finally inverted. Then, my mathematical expectation will be $(p_1 - p_2)^n$.

The difference $p_1 - p_2$ is certainly less than 1; so that if n is very great my expectation will be zero; we need not learn p_1 and p_2 to be aware that the game is equitable.

There would always be an exception if one of the numbers p_1 and p_2 was equal to 1 and the other naught. *Then it would not apply because our initial hypotheses would be too simple.*

What we have just seen applies not only to the mixing of cards but to all mixings, to those of powders and of liquids; and even to those of the molecules of gases in the kinetic theory of gases.

To return to this theory, suppose for a moment a gas whose molecules cannot mutually clash, but may be deviated by hitting the insides of the vase wherein the gas is confined. If the form of the vase is sufficiently complex the distribution of the molecules and that of the velocities will not be long in becoming uniform. But this will not

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be so if the vase is spherical or if it has the shape of a cuboid. Why? Because in the first case the distance from the center to any trajectory will remain constant; in the second case this will be the absolute value of the angle of each trajectory with the faces of the cuboid.

So we see what should be understood by conditions *too simple*; they are such as conserve something, which leave an invariant remaining. Are the differential equations of the problem too simple for us to apply the laws of chance? This question would seem at first view to lack precise meaning; now we know what it means. They are too simple if they conserve something, if they admit a uniform integral. If something in the initial conditions remains unchanged, it is clear the final situation can no longer be independent of the initial situation.

We come finally to the theory of errors. We know not to what are due the accidental errors, and precisely because we do not know we are aware they obey the law of Gauss. Such is the paradox. The explanation is nearly the same as in the preceding cases. We need know only one thing: that the errors are very numerous, that they are very slight, that each may be as well negative as positive. What is the curve of probability of each of them? We do not know; we only suppose it is symmetric. We prove then that the resultant error will follow Gauss's law, and this resulting law is independent of the particular laws which we do not know. Here again the simplicity of the result is born of the very complexity of the data.

* * *

But we are not through with paradoxes. I have just recalled the figment of Flammarion, that of the man going quicker than light, for whom time changes sign. I said that for him all phenomena would seem due to chance. That is true from a certain point of view, and yet all these

phenomena at a given moment would not be distributed in conformity with the laws of chance since the distribution would be the same as for us, who seeing them unfold harmoniously and without coming out of a primal chaos, do not regard them as ruled by chance.

What does that mean? For Lumen, Flammarion's man, slight causes seem to produce great effects; why do not things go on as for us when we think we see grand effects due to little causes? Would not the same reasoning be applicable in his case?

Let us return to the argument. When slight differences in the causes produce vast differences in the effects, why are these effects distributed according to the laws of chance? Suppose a difference of a millimeter in the cause produces a difference of a kilometer in the effect. If I win in case the effect corresponds to a kilometer bearing an even number, my probability of winning will be 1/2. Why? Because to make that, the cause must correspond to a millimeter with an even number. Now, according to all appearance, the probability of the cause varying between certain limits will be proportional to the distance apart of these limits, provided this distance be very small. If this hypothesis were not admitted there would no longer be any way of representing the probability by a continuous function.

What now will happen when great causes produce small effects? This is the case where we should not attribute the phenomenon to chance and where on the contrary Lumen would attribute it to chance. To a difference of a kilometer in the cause would correspond a difference of a millimeter in the effect. Would the probability of the cause being comprised between two limits n kilometers apart still be proportional to n? We have no reason to suppose so, since this distance, n kilometers, is great. But the probability that the effect lies between two limits n

millimeters apart will be precisely the same, so it will not be proportional to *n*, even though this distance, *n* millimeters, be small. There is no way therefore of representing the law of probability of effects by a continuous curve. This curve, understand, may remain continuous in the *analytic* sense of the word; to *infinitesimal* variations of the abscissa will correspond infinitesimal variations of the ordinate. But *practically* it will not be continuous, since *very small* variations of the ordinate would not correspond to very small variations of the abscissa. It would become impossible to trace the curve with an ordinary pencil; that is what I mean.

So what must we conclude? Lumen has no right to say that the probability of the cause (*his* cause, our effect) should be represented necessarily by a continuous function. But then why have we this right? It is because this state of unstable equilibrium which we have been calling initial is itself only the final outcome of a long previous history. In the course of this history complex causes have worked a great while: they have contributed to produce the mixture of elements and they have tended to make everything uniform at least within a small region; they have rounded off the corners, smoothed down the hills and filled up the valleys. However capricious and irregular may have been the primitive curve given over to them, they have worked so much toward making it regular that finally they deliver over to us a continuous curve. And this is why we may in all confidence assume its continuity.

Lumen would not have the same reasons for such a conclusion. For him complex causes would not seem agents of equalization and regularity, but on the contrary would create only inequality and differentiation. He would see a world more and more varied come forth from a sort of primitive chaos. The changes he could observe would be for him unforeseen and impossible to foresee.

They would seem to him due to some caprice or another; but this caprice would be quite different from our chance, since it would be opposed to all law, while our chance still has its laws. All these points call for lengthy explications which perhaps would aid in the better comprehension of the irreversibility of the universe.

* * *

We have sought to define chance, and now it is proper to put a question. Has chance thus defined, in so far as this is possible, objectivity?

It may be questioned. I have spoken of very slight or very complex causes. But what is very little for one may be very big for another, and what seems very complex to one may seem simple to another. In part I have already answered by saying precisely in what cases differential equations become too simple for the laws of chance to remain applicable. But it is fitting to examine the matter a little more closely, because we may take still other points of view.

What means the phrase "very slight"? To understand it we need only go back to what has already been said. A difference is very slight, an interval is very small, when within the limits of this interval the probability remains sensibly constant. And why may this probability be regarded as constant within a small interval? It is because we assume that the law of probability is represented by a continuous curve, continuous not only in the analytic sense but *practically* continuous, as already explained. This means that it not only presents no absolute hiatus but that it has neither salients nor reentrants too acute or too accentuated.

And what gives us the right to make this hypothesis? We have already said it is because, since the beginning of the ages, there have always been complex causes ceaselessly acting in the same way and making the world tend

toward uniformity without ever being able to turn back. These are the causes which little by little have flattened the salients and filled up the reentrants and this is why our probability curves now show only gentle undulations. In milliards of milliards of ages another step will have been made toward uniformity, and these undulations will be ten times as gentle; the radius of mean curvature of our curve will have become ten times as great. And then such a length as seems to us to-day not very small, since on our curve an arc of this length cannot be regarded as rectilineal, should on the contrary at that epoch be called very little, since the curvature will have become ten times less and an arc of this length may be sensibly identified with a sect.

Thus the phrase "very slight" remains relative; but it is not relative to such or such a man, it is relative to the actual state of the world. It will change its meaning when the world shall have become more uniform, when all things shall have blended still more. But then doubtless men can no longer live and must give place to other beings—should I say far smaller or far larger? So that our criterion, remaining true for all men, retains an objective sense.

And on the other hand what means the phrase "very complex"? I have already given one solution, but there are others. Complex causes we have said produce a blend more and more intimate, but after how long a time will this blend satisfy us? When will it have accumulated sufficient complexity? When shall we have sufficiently shuffled the cards? If we mix two powders, one blue the other white, there comes a moment when the tint of the mixture seems to us uniform because of the feebleness of our senses; it will be uniform for the presbyte, forced to gaze from afar, before it will be so for the myope. And when it has become uniform for all eyes, we still could push back the limit by the use of instruments. There is

no chance for any man ever to discern the infinite variety which, if the kinetic theory is true, hides under the uniform appearance of a gas. And yet if we accept Gouy's ideas on the Brownian movement, does not the microscope seem on the point of showing us something analogous?

This new criterion is therefore relative like the first; and if it retains an objective character, it is because all men have approximately the same senses, the power of their instruments is limited, and besides they use it only exceptionally.

* * *

It is just the same in the moral sciences and particularly in history. The historian is obliged to make a choice among the events of the epoch he studies; he recounts only those which seem to him the most important. He therefore contents himself with relating the most momentous events of the sixteenth century for example, as likewise the most remarkable facts of the seventeenth century. If the first suffice to explain the second, we say these conform to the laws of history. But if a great event of the seventeenth century should have for cause a small fact of the sixteenth century which no history reports, which all the world has neglected, then we say this event is due to chance. This word has therefore the same sense as in the physical sciences; it means that slight causes have produced great effects.

The greatest bit of chance is the birth of a great man. It is only by chance that meeting of two germinal cells, of different sex, containing precisely, each on its side, the mysterious elements whose mutual reaction must produce the genius. One will agree that these elements must be rare and that their meeting is still more rare. How slight a thing it would have required to deflect from its route the carrying spermatozoon. It would have sufficed to deflect it a tenth of a millimeter and Napoleon would not have

been born and the destinies of a continent would have been changed. No example can better make us understand the veritable characteristics of chance.

One more word about the paradoxes brought out by the application of the calculus of probabilities to the moral sciences. It has been proved that no Chamber of Deputies will ever fail to contain a member of the opposition, or at least such an event would be so improbable that we might without fear wager the contrary, and bet a million against a sou.

Condorcet has striven to calculate how many jurors it would require to make a judicial error practically impossible. If we had used the results of this calculation, we should certainly have been exposed to the same disappointments as in betting, on the faith of the calculus, that the opposition would never be without a representative.

The laws of chance do not apply to these questions. If justice be not always meted out to accord with the best reasons, it uses less than we think the method of Bridoye. This is perhaps to be regretted, for then the system of Condorcet would shield us from judicial errors.

What is the meaning of this? We are tempted to attribute facts of this nature to chance because their causes are obscure; but this is not true chance. The causes are unknown to us it is true, and they are even complex; but they are not sufficiently so, since they conserve something. We have seen that this it is which distinguishes causes "too simple." When men are brought together they no longer decide at random and independently one of another; they influence one another. Multiplex causes come into action. They worry men, dragging them to right or left, but one thing there is they cannot destroy, this is their Panurge flock-of-sheep habits. And this is an invariant.

* * *

Difficulties are indeed involved in the application of the

calculus of probabilities to the exact sciences. Why are the decimals of a table of logarithms, why are those of the number π distributed in accordance with the laws of chance? Elsewhere I have already studied the question in so far as it concerns logarithms, and there it is easy. It is clear that a slight difference of argument will give a slight difference of logarithm, but a great difference in the sixth decimal of the logarithm. Always we find again the same criterion.

But as for the number π, that presents more difficulties, and I have at the moment nothing worth while to say.

There would be many other questions to resolve, had I wished to attack them before solving that which I more specially set myself. When we reach a simple result, when we find for example a round number, we say that such a result cannot be due to chance, and we seek, for its explanation, a non-fortuitous cause. And in fact there is only a very slight probability that among 10,000 numbers chance will give a round number, for example the number 10,000. This has only one chance in 10,000. But there is only one chance in 10,000 for the occurrence of any other one number; and yet this result will not astonish us, nor will it be hard for us to attribute it to chance; and that simply because it will be less striking.

Is this a simple illusion of ours, or are there cases where this way of thinking is legitimate? We must hope so, else were all science impossible. When we wish to check a hypothesis, what do we do? We cannot verify all its consequences, since they would be infinite in number; we content ourselves with verifying certain ones and if we succeed we declare the hypothesis confirmed, because so much success could not be due to chance. And this is always at bottom the same reasoning.

I cannot completely justify it here, since it would take too much time; but I may at least say that we find our-

selves confronted by two hypotheses, either a simple cause or that aggregate of complex causes we call chance. We find it natural to suppose that the first should produce a simple result, and then, if we find that simple result, the round number for example, it seems more likely to us to be attributable to the simple cause which must give it almost certainly, than to chance which could only give it once in 10,000 times. It will not be the same if we find a result which is not simple; chance, it is true, will not give this more than once in 10,000 times; but neither has the simple cause any more chance of producing it.

HENRI POINCARÉ.

PARIS, FRANCE.

THE THEORY OF REVERSIONS.*

SQUARES like those shown in Figs. 1 and 2, in which the numbers occur in their natural order, are known as *natural squares.* In such squares, it will be noticed that the numbers in associated[1] cells are complementary, i. e., their sum is twice the mean number. It follows that any two columns equally distant from the central bar of the lattice are complementary columns, that is, the magic sum

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Fig. 1.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Fig. 2.

will be the mean of their sums. Further any two numbers in these complementary columns which lie in the same row have a constant difference, and therefore the sums of the two columns differ by n times this difference. If then we raise the lighter column and depress the heavier column by $n/2$ times this difference we shall bring both to the

* This paper was extracted about 18 months ago from three different parts of an unpublished treatise written in 1894. With regard to footnote 6, p. 63, since this was written Sayles and Worthington have independently solved the problem of construction for 6^2.

[1] Two cells are said to be *associated* when the straight line joining their centers intersects the center of the lattice, and they are equally distant from that center.

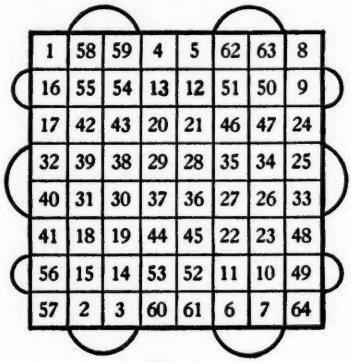
mean value. Now we can effect this change by interchanging half the numbers in the one column with the numbers in the other column lying in their respective rows. The same is true with regard to rows, so that if we can make $n/2$ horizontal interchanges between every pair of complementary columns and the same number of vertical interchanges between every pair of complementary rows, we shall have the magic sum in all rows and columns. It is easy to see that we can do this by reversing half the rows and half the columns, provided the two operations are so arranged as not to interfere with one another. This last condition can be assured by always turning over columns and rows in associated pairs, for then we shall have made horizontal interchanges only between pairs of numbers previously untouched or between pairs, each of whose constituents has already received an equal vertical displacement; and similarly with the vertical interchanges. By this method, it will be noticed, we always secure magic central diagonals, for however we choose our rows and columns we only alter the central diagonals of the natural square (which are already magic) by interchanging pairs of complementaries with other pairs of complementaries.

Since the $n/2$ columns have to be arranged in pairs on either side of the central vertical bar of the lattice, $n/2$ must be even, and so the method, *in its simplest form,* applies only to orders $\equiv 0 \pmod 4$. We may formulate the rule thus: *For orders of form 4m, reverse m pairs of complementary columns and m pairs of complementary rows, and the crude magic is completed.*

In the following example the curved lines indicate the rows and columns which have been reversed (Fig. 3).

We have said that this method applies only when $n/2$ is even, but we shall now show that by a slight modification it can be applied to all even orders. For suppose n is double-of-odd; we cannot then arrange half the columns

in pairs about the center since their number is odd, but we can so arrange $n/2$—1 rows and $n/2$—1 columns, and if we reverse all these rows and columns we shall have made $n/2$—1 interchanges between every pair of complementary rows and columns. We now require only to make the one further interchange between every pair of rows and columns, without interfering with the previous changes or with the central diagonals. To effect this is always



1	58	59	4	5	62	63	8
16	55	54	13	12	51	50	9
17	42	43	20	21	46	47	24
32	39	38	29	28	35	34	25
40	31	30	37	36	27	26	33
41	18	19	44	45	22	23	48
56	15	14	53	52	11	10	49
57	2	3	60	61	6	7	64

Fig. 3.

easy with any orders $\equiv 2 \pmod 4$, (6, 10, 14 etc.), excepting the first. In the case of 6^2 an artifice is necessary. If we reverse the two central diagonals of a square it will be found, on examination, that this is equivalent to reversing two rows and two columns; in fact, this gives us a method of forming the magic 4^2 from the natural square with the least number of displacements, thus:

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Fig. 4.

Applying this idea, we can complete the crude magic

6^2 from the scheme shown in Fig. 5, where horizontal lines indicate horizontal interchanges, and vertical lines vertical interchanges; the lines through the diagonals implying that the diagonals are to be reversed. The resulting magic is shown in Fig. 6.

The general method here described is known as the *method of reversions*, and the artifice used in the double-of-odd orders is called *the broken reversion*. The method of reversions, as applied to all even orders, both in squares and cubes, was first(?) investigated by the late W. Firth, Scholar of Emmanuel, Cambridge.[2]

The broken reversion for 6^2 may, of course, be made in various ways, but the above scheme is one of the most sym-

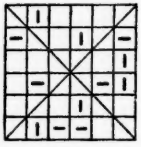


Fig. 5.

36	32	3	4	5	31
12	29	9	28	26	7
13	14	22	21	17	24
19	23	16	15	20	18
25	11	27	10	8	30
6	2	34	33	35	1

Fig. 6.

metrical, and may be memorialized thus: *For horizontal changes commence at the two middle cells of the bottom row, and progress upwards and divergently along two knight's paths. For vertical changes turn the square on one of its sides and proceed as before.*

In dealing with larger double-of-odd orders we may leave the central diagonals "intact" and invert $n/2-1$ rows and $n/2-1$ columns. The broken reversion can then always be effected in a multitude of ways. It must be kept in mind, however, that in making horizontal changes we must not touch numbers which have been already moved horizontally, and if we use a number which has received

[2] Died 1889. For historical notice *vide* section on cubes.

a vertical displacement we can only change it with a number which has received an equal vertical displacement, and similarly with vertical interchanges. Lastly we must not touch the central diagonals.

Fig. 7 is such a scheme for 10^2, with the four central rows and columns reversed, and Fig. 8 shows the completed magic.

It is unnecessary to formulate a rule for making the reversions in these cases, because we are about to consider the method from a broader standpoint which will lead up to a general rule.

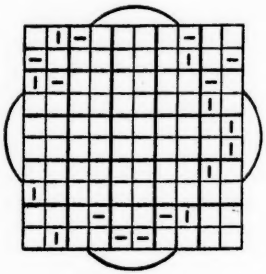


Fig. 7.

1	92	8	94	95	96	97	3	9	10
20	12	13	84	85	86	87	88	19	11
71	29	23	74	75	76	77	28	22	30
40	39	38	67	66	65	64	33	62	31
50	49	48	57	56	55	54	43	42	51
60	59	58	47	46	45	44	53	52	41
70	69	68	37	36	35	34	63	32	61
21	72	73	24	25	26	27	78	79	80
81	82	83	17	15	16	14	18	89	90
91	2	93	4	6	5	7	98	99	100

Fig. 8.

If the reader will consider the method used in forming the magic 6^2 by reversing the central diagonals, he will find that this artifice amounts to taking in every column two numbers equally distant from the central horizontal bar and interchanging each of them with its complementary in the associated cell, the operation being so arranged that two and only two numbers are moved in each row. This, as we have already pointed out, is equivalent to reversing two rows and two columns. Now these skew interchanges need not be made on the central diagonals—they can be made in any part of the lattice, provided the con-

ditions just laid down are attended to. If then we make a second series of skew changes of like kind, we shall have, in effect, reversed 4 rows and 4 columns, and so on, each complete skew reversion representing two rows and columns. Now if $n \equiv 2 \pmod 4$ we have to reverse $n/2-1$ rows and colunms before making the broken reversion, therefore the same result is attained by making $(n-2)/4$ complete sets of skew reversions and one broken reversion.

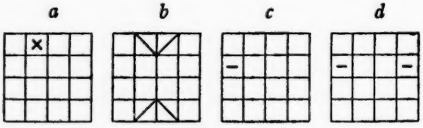


Fig. 9.

In like manner, if $n \equiv 0 \pmod 4$, instead of reversing $n/2$ rows and columns we need only to make $n/4$ sets of skew reversions.

We shall define the symbol [×] as implying that skew interchanges are to be made between opposed pairs of the four numbers symmetrically situated with regard to the central horizontal and vertical bars, one of which numbers

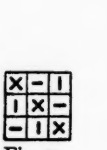


Fig. 10.

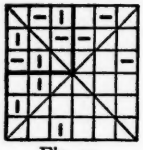


Fig. 11.

36	5	33	4	2	31
25	29	10	9	26	12
18	20	22	21	17	13
19	14	16	15	23	24
7	11	27	28	8	30
6	32	3	34	35	1

Fig. 12.

occupies the cell in which the symbol is placed. In other words we shall assume that Fig. 9*a* indicates what we have hitherto represented as in Fig. 9*b*. Further, it is quite unnecessary to use two symbols for a vertical or horizontal change, for Fig. 9*c* sufficiently indicates the same as Fig. 9*d*. If these abbreviations are granted, a scheme like Fig.

5 may be replaced by a small square like Fig. 10, which is to be applied to the top left-hand corner of the natural 6^2.

Fig. 11 is the extended scheme from Fig. 10, and Fig. 12 is the resulting magic. The small squares of symbols like Fig. 10 may be called *"index squares."*

The law of formation for index squares is sufficiently obvious. To secure magic rows and columns in the resulting square, the symbols — and | must occur once on each row and column of the index, and the symbol × an equal number of times on each row and column; that is, if there are two series $\times\times\ldots.\times$ the symbol × must appear twice in every row and twice in every column, and so on. But we already know by the theory of paths that these conditions can be assured by laying the successive symbolic periods along parallel paths of the index, whose coordinates are prime to the order of the index. If we decide always to use parallel diagonal paths and always to apply the index to the top left-hand corner of the natural square, the index square will be completely represented by its top row. In Fig. 10 this is |×|−|1|, which we may call the index-rod of the square, or we may simply call Fig. 12 the magic |×|−|1|. Remembering that we require $(n—2)/4$ sets of skew reversions when $n \equiv 2 \pmod 4$ and $n/4$ when $n \equiv 0$, it is obvious that the following rule will give crude magic squares of any even order n:

Take a rod of $n/2$ cells, $n/4$ symbols of the form ×, (using the integral part of $n/4$ only), and if there is a remainder when n is divided by 4, add the symbols | and —. Place one of the symbols × in the left-hand cell of the rod, and the other symbols in any cell, but not more than one in each cell. The result is an index-rod for the magic n^2.

Take a square lattice of order $n/2$, and lay the rod along the top row of the lattice. Fill up every diagonal slanting downward and to the right which has a symbol in its highest cell with repetitions of that symbol. The re-

sulting index-square if applied to the lop left-hand corner of the natural n^2, with the symbols allowed the operative powers already defined, will produce the magic n^2.

The following are index-rods for squares of even orders:

4^2 [×][ ]

6^2 [×][−][ ]

8^2 [×][ ][×][ ]

10^2 [×][ ][ ][×][−]

12^2 [×][ ][ ][×][×][ ]

14^2 [×][−][×][ ][ ][×][ ]

When the number of cells in the rod exceeds the number of symbols, as it always does excepting with 6^2, the first cell may be left blank. Also, if there are sufficient blank cells, a × may be replaced by two vertical and two horizontal symbols. Thus 12^2 might be given so [×][ ][ ][−][×][−]

[×][ ][ ][−][×][−]

×	|	|	−	×	−						
−	×	|	|	−	×						
×	−	×	|	|	−						
−	×	−	×	|	|						
|	−	×	−	×	|						
|	|	−	×	−	×						

Fig. 13.

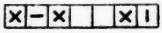
144	134	135	9	140	7	6	137	4	10	11	133
24	131	123	124	20	127	126	17	21	22	122	13
120	35	118	112	113	31	30	32	33	111	26	109
48	107	46	105	101	102	43	44	100	39	98	37
85	59	94	57	92	90	55	89	52	87	50	60
73	74	70	81	68	79	78	65	76	63	71	72
61	62	75	69	77	67	66	80	64	82	83	84
49	86	58	88	56	54	91	53	93	51	95	96
97	47	99	45	41	42	103	104	40	106	38	108
36	110	34	28	29	114	115	116	117	27	119	25
121	23	15	16	125	19	18	128	129	130	14	132
12	2	3	136	8	138	139	5	141	142	143	1

Fig. 14.

This presentation of 12^2 is shown in Figs. 13, 14, and 14^2 from the index-rod given above, in Figs. 15, 16.

Of course the employment of diagonal paths in the construction of the index is purely a matter of convenience. In the following index for 10^2, (Fig. 17) the skew-symbols

are placed along two parallel paths (2, 1) and the symbols — and | are then added so that each shall appear once in each row and once in each column, but neither of them on the diagonal of the index slanting upward and to the left.



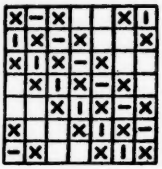


Fig. 15.

196	13	194	4	5	191	189	8	188	10	11	185	2	183
169	181	26	179	19	20	176	175	23	24	172	17	170	28
168	156	166	39	164	34	35	36	37	159	32	157	41	155
43	153	143	151	52	149	49	50	146	47	144	54	142	56
57	58	138	130	136	65	134	133	62	131	67	129	69	70
126	72	73	123	117	121	78	77	118	80	116	82	83	113
98	111	87	88	108	104	106	105	93	103	95	96	100	85
99	97	101	102	94	90	92	91	107	89	109	110	86	112
84	114	115	81	75	79	119	120	76	122	74	124	125	71
127	128	68	60	66	132	64	63	135	61	137	59	139	140
141	55	45	53	145	51	147	148	48	150	46	152	44	154
42	30	40	158	38	160	161	162	163	33	165	31	167	29
15	27	171	25	173	174	22	21	177	178	18	180	16	182
14	184	12	186	187	9	7	190	6	192	193	3	195	1

Fig. 16.

Crude cubes of even orders we shall treat by the index-rod as in the section on squares. The reader will remember that we constructed squares of orders $\equiv 0 \pmod 4$ by re-

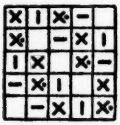
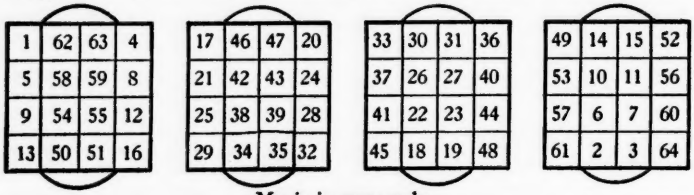


Fig. 17.

versing half the rows and half the columns, and it is easy to obtain an analogous method for the cubes of the same family. Suppose we reverse half the V-planes[3] in asso-

[3] P-plane = Presentation-, or Paper-plane; H-plane = Horizontal plane; V-plane = Vertical plane.

ciated pairs; that is, turn each through an angle of 180° round a horizontal axis parallel to the paper-plane so that the associated columns in each plane are interchanged and reversed. We evidently give to every row of the cube the magic sum, for half the numbers in each row will be ex-



1	62	63	4
5	58	59	8
9	54	55	12
13	50	51	16

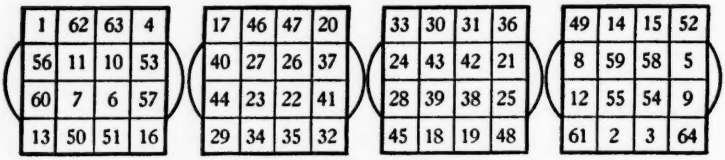
17	46	47	20
21	42	43	24
25	38	39	28
29	34	35	32

33	30	31	36
37	26	27	40
41	22	23	44
45	18	19	48

49	14	15	52
53	10	11	56
57	6	7	60
61	2	3	64

Magic in rows only.

Fig. 18. The natural 4^3 with V-planes reversed.



1	62	63	4
56	11	10	53
60	7	6	57
13	50	51	16

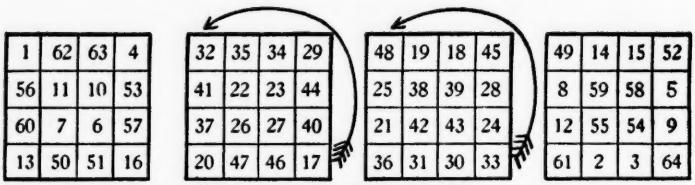
17	46	47	20
40	27	26	37
44	23	22	41
29	34	35	32

33	30	31	36
24	43	42	21
28	39	38	25
45	18	19	48

49	14	15	52
8	59	58	5
12	55	54	9
61	2	3	64

Magic in rows and columns.

Fig. 19. Being Fig. 18 with H-planes reversed.



1	62	63	4
56	11	10	53
60	7	6	57
13	50	51	16

32	35	34	29
41	22	23	44
37	26	27	40
20	47	46	17

48	19	18	45
25	38	39	28
21	42	43	24
36	31	30	33

49	14	15	52
8	59	58	5
12	55	54	9
61	2	3	64

Magic in rows, columns and lines.

Fig. 20. Being Fig. 19, with P-planes reversed.

CRUDE MAGIC 4^3.

changed for their complementaries. If we do likewise with H-planes and P-planes the rows and lines[4] will become magic. But as with the square, and for like reasons, these three operations can be performed without mutual interference. Hence the simple general rule for all cubes of the double-of-even orders:

[4] "Line" = a contiguous series of cells measured at right angles to the paper-plane.

Reverse, in associated pairs, half the V-planes, half the H-planes, and half the P-planes.

With this method the central great diagonals, of course, maintain their magic properties, as they must do for the cube to be considered even a crude magic.[5] To make the operation clear to the reader we append views of 4^3 at each

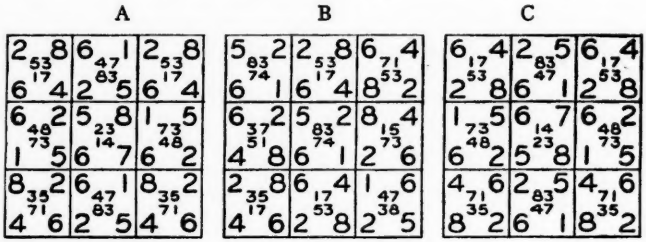


Fig. 21.

separate stage, the central pair of planes being used at each reversion.

By this method the reader can make any crude magic cube of order $4m$. With orders of form $4m+2$ we find the same difficulties as with squares of like orders. So far as we are aware no magic cube of this family had been

1	17	24
23	3	16
18	22	2

15	19	8
7	14	21
20	9	13

26	6	10
12	25	5
4	11	27

Fig. 22.

constructed until Firth succeeded with 6^3 in 1889, and we believe those we shall presently construct are the first which have been published.[6] Firth's original cube was built up by the method of "pseudo-cubes," being an extension to solid magics of Thompson's method. The cube of 216 cells was divided into 27 subsidiary cubes each con-

[5] A cube which is faulty on one of its central great diagonals is no more a magic than is a square which is faulty on one of its central diagonals.

[6] The recent examples published by Willis and Kingery fail in their central great diagonals, a fatal defect.

taining 2 cells in an edge. The 8 cells of each subsidiary were filled with the numbers 1 to 8 in such a way that each row, column, line, *and central great diagonal* of the large cube summed 27. The cube was then completed by using the magic 3^3 in the same way that 6^2 is constructed from 3^2. Firth formulated no rule for arrangement of the numbers in the pseudo-cubes, and great difficulty was encountered in balancing the central great diagonals. His pseudo-

I

2	8	134	129	186	192
6	4	130	133	190	188
182	178	21	24	121	125
177	181	22	23	126	122
144	138	174	169	16	10
140	142	170	173	12	14

II

5	3	132	135	189	187
1	7	136	131	185	191
180	184	18	19	127	123
183	179	17	20	124	128
139	141	172	175	11	13
143	137	176	171	15	9

III

117	114	146	152	62	60
118	113	150	148	64	58
54	50	109	106	168	164
52	56	110	105	162	166
154	160	70	68	97	102
156	158	66	72	98	101

IV

120	115	149	147	63	59
119	116	145	151	61	59
51	55	112	107	161	165
53	49	111	108	167	163
155	157	65	71	100	103
153	159	69	67	99	104

V

206	204	42	45	78	76
202	208	46	41	74	80
89	93	198	199	38	34
94	90	197	200	33	37
28	30	82	85	212	214
32	26	86	81	216	210

VI

201	207	48	43	73	79
205	203	44	47	77	75
95	91	193	196	36	40
92	96	194	195	39	35
31	25	88	83	215	209
27	29	84	87	211	213

Fig. 23.

skeleton is shown in Fig. 21, where each plate represents two P-planes of 6^3, each plate containing 9 pseudo-cubes. The numbers in the subsidiaries are shown in diagrammatic perspective, the four "larger" numbers lying in the anterior layer, and the four "smaller" numbers, grouped in the center, in the posterior layer.

If we use this with the magic of Fig. 22 we obtain the magic 6^3 shown in Fig. 23.

This cube is non-La Hireian, as is frequently the case with magics constructed by this method.

The scheme of pseudo-cubes for 6^3 once found, we can easily extend the method to any double-of-odd order in the following manner. Take the pseudo-scheme of next lower order [e. g., 6^3 to make 10^3, 10^3 to make 14^3 etc.]. To each of three outside plates of cubes, which meet at any corner of the skeleton, apply a replica-plate, and to each of the other three faces a complementary to the plate opposed to it, that is a plate in which each number replaces its complementary number (1 for 8, 2 for 7, etc.). We now have a properly balanced skeleton for the next double-of-odd order, wanting only its 12 edges. Consider any three edges that meet at a corner of the cube; they can be completed (wanting their corner-cubes) by placing in each of them any row of cubes from the original skeleton. Each of these three edges has three other edges parallel to it, two lying in the same square planes with it and the third diagonally opposed to it. In the former we may place edges complementary to the edge to which they are parallel, and in the latter a replica of the same. The skeleton wants now only its 8 corner pseudo-cubes. Take any cube and place it in four corners, no two of which are in the same row, line, column, or great diagonal (e. g. B, C. E, H in Fig. 38), and in the four remaining corners place its complementary cube. The skeleton is now complete, and the cube may be formed from the odd magic of half its order.

This method we shall not follow further, but shall now turn to the consideration of index-cubes, an artifice far preferable.

Before proceeding the reader should carefully study the method of the index-rod as used for magic squares (pp. 57-61).

The reversion of a pair of planes in each of the three

aspects, as previously employed for 4^3, is evidently equivalent to interchanging two numbers with their complementaries in every row, line, and column of the natural cube. If therefore we define the symbol $\times$ as implying that such an interchange is to be made not only from the cell in which it is placed, but also from the three other cells with which it is symmetrically situated in regard to the central horizontal and vertical bars of its P-plane, and can make

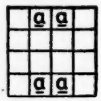


Fig. 24.

one such symbol operate in every row, line and column of an index-cube whose edge is half that of the great cube, we shall have secured the equivalent of the above-mentioned reversion. For example, a $\times$ placed in the second cell of the top row of any P-plane of 4^3, will denote that the four numbers marked *a* in Fig. 24 are each to be interchanged with its complement, which lies in the associated cell in the associated P-plane.

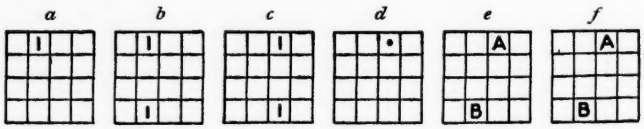


FIG. 25.

From this it follows that we shall have a complete reversion scheme for any order $4m$, by placing in every row, line and column of the index $(2m)^3$, m of the symbols $\times$. In the case of orders $4m+2$, after placing m such symbols in the cube $(2m+1)^3$, we have still to make the equivalent of one reversed plane in each of the three aspects. This amounts to making one symmetrical vertical interchange, one symmetrical horizontal interchange, and one

symmetrical interchange at right angles to the paper-plane in every row, line and column. If we use the symbol | to denote such a vertical interchange, not only for the cell in which it stands, but also for the associated cell, and give like meanings to — and ·, for horizontal changes and changes along lines, we shall have made the broken reversion when we allow each of these symbols to operate once in every row, column and line of the index. For example, *a* in Fig. 25 means *b* in its own P-plane, and *c* in the associated P-plane; while *d* indicates that the numbers lying in its own P-plane as in *e* are to be interchanged, A with A and B with B, with the numbers lying in the associated plane *f*. We can always prepare the index, provided the rod does not contain a less number of cells than the number of symbols, by the following rule, n being the order.

Take an index-rod of $n/2$ cells, $n/4$ symbols of the form $\times$, (using the integral part of $n/4$ only), and if there is any remainder when n is divided by 4 add the three symbols |, —, ·. Now prepare an index square in the way described on p. 59, but using the diagonals upward and to the right instead of upward to the left,[7] and take this square as the first P-plane of an index-cube. Fill every *great* diagonal of the cube, running to the *right, down* and *away,* which has a symbol in this P-plane cell, with repetitions of that symbol.[8] This index-cube applied to the near, left-hand, top corner of the natural n^3, with the symbols allowed the operative powers already defined, will make the magic n^3.

This method for even orders applies universally with the single exception of 6^3, and in the case of 6^3 we shall presently show that the broken reversion can still be made

[7] Either way will do, but it happens that the former has been used in the examples which follow.

[8] More briefly, in the language of Paths, the symbols are laid, in the square, on (1,1); their repetitions in the cube, on (1, —1, 1).

by scattering the symbols over the whole cube. The following are index-rods for various cubes.

4^3 [×][ ]

8^3 [×][ ][ ][×]

10^3 [×][ı][−][×][·]

12^3 [ ][ ][×][×][ ][×]

14^3 [ ][×][−][×][·][×][ı]

As in the case of index-rods for squares, the first cell may be left blank, otherwise it must contain a ×.

I			
64	2	3	61
5	59	58	8
9	55	54	12
52	14	15	49

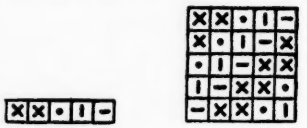
II			
48	18	19	45
21	43	42	24
25	39	38	28
36	30	31	33

III			
32	34	35	29
37	27	26	40
41	23	22	44
20	46	47	17

IV			
16	50	51	13
53	11	10	56
57	7	6	60
4	62	63	1

Fig. 26.

Fig. 26 is a 4^3, made with the index-rod given above. It has only half the numbers removed from their natural places. Figs. 27 and 28 are the index-rod, index-square and index-cube for 10^3, and Fig. 29 is the extended reversion scheme obtained from these, in which \ and / denote single changes between associated cells, and the symbols |, —, and ·, single changes parallel to columns, rows, and lines. Figs. 30 and 31 show the resulting cube.



Index Rod. Index Square.

Fig. 27.

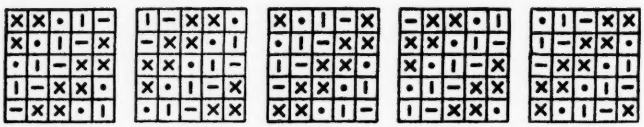


Fig. 28. Index Cube.

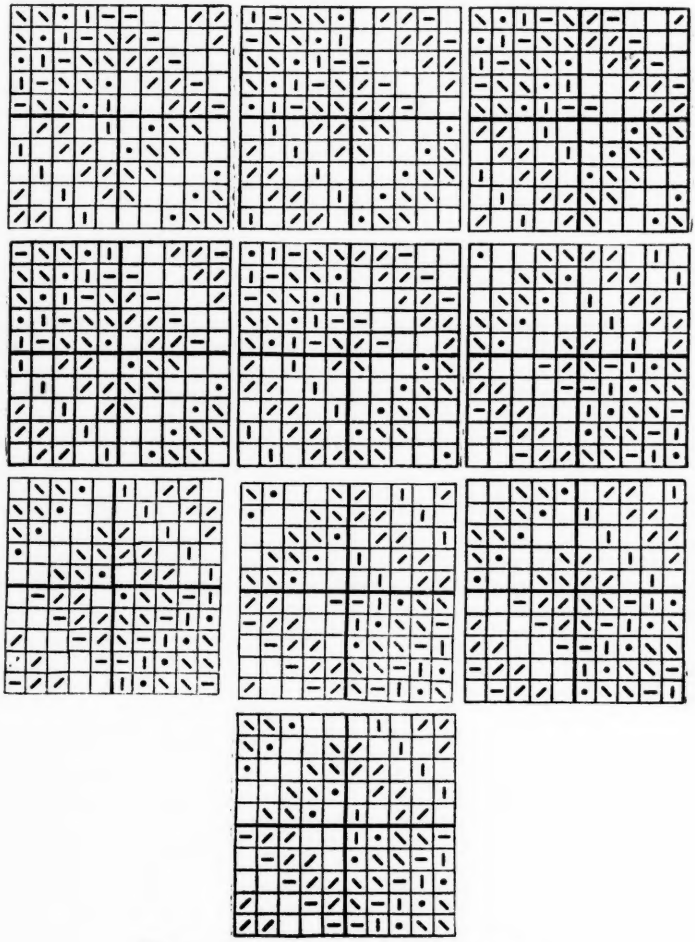


Fig. 29. Extended Reversion Scheme for 10^5.

1000	999	903	94	6	5	7	8	992	991
990	912	83	17	986	985	14	18	19	981
921	72	28	977	976	975	974	23	29	30
61	39	968	967	935	36	964	963	32	40
50	959	958	944	55	46	47	953	952	41
51	949	948	54	45	56	957	943	942	60
31	62	938	937	65	966	934	933	69	70
71	22	73	927	926	925	924	78	79	980
920	82	13	84	916	915	87	88	989	911
910	909	93	4	95	96	97	998	902	901

191	109	898	897	805	106	894	893	102	110
120	889	888	814	185	116	117	883	882	111
880	879	823	174	126	125	127	128	872	871
870	832	163	137	866	865	134	138	139	861
841	152	148	857	856	855	854	143	149	150
151	142	153	847	846	845	844	158	159	860
840	162	133	164	836	835	167	168	869	831
830	829	173	124	175	176	177	878	822	821
181	819	818	184	115	186	887	813	812	190
101	192	808	807	195	896	804	803	199	200

800	702	293	207	796	795	204	208	209	791
711	282	218	787	786	785	784	213	219	220
271	229	778	777	725	226	774	773	222	230
240	769	768	734	265	236	237	763	762	231
760	759	743	254	246	245	247	248	752	751
750	749	253	244	255	256	257	758	742	741
261	739	738	264	235	266	767	733	732	270
221	272	728	727	275	776	724	723	279	280
281	212	283	717	716	715	714	288	289	790
710	292	203	294	706	705	297	298	799	701

310	699	698	604	395	306	307	693	692	301
690	689	613	384	316	315	317	318	682	681
680	622	373	327	676	675	324	328	329	671
631	362	338	667	666	665	664	333	339	340
351	349	658	657	645	346	654	653	342	350
341	352	648	647	355	656	644	643	359	360
361	332	363	637	636	635	634	368	369	670
630	372	323	374	626	625	377	378	679	621
620	619	383	314	385	386	387	688	612	611
391	609	608	394	305	396	697	603	602	400

501	492	408	597	596	595	594	403	409	410
481	419	588	587	515	416	584	583	412	420
430	579	578	524	475	426	427	573	572	421
570	569	533	464	436	435	437	438	562	561
560	542	453	447	556	555	444	448	449	551
550	452	443	454	546	545	457	458	559	541
540	539	463	434	465	466	467	568	532	531
471	529	528	474	425	476	577	523	522	480
411	482	518	517	485	586	514	513	489	490
491	402	493	507	506	505	504	498	499	600

401	502	503	497	496	495	494	508	599	510
511	512	488	487	415	516	484	483	519	590
521	479	478	424	525	576	527	473	472	530
470	469	433	534	535	536	567	538	462	461
460	442	543	544	456	455	547	558	549	451
450	552	553	557	446	445	554	548	459	441
440	439	563	564	566	565	537	468	432	431
580	429	428	574	575	525	477	423	422	571
581	589	418	417	585	486	414	413	582	520
591	592	598	407	406	405	404	593	509	500

Fig. 30. First 6 plates of 10^3, made from Fig. 29. (Sum = 5005.)

601	399	398	304	605	696	607	393	392	610
390	389	313	614	615	616	687	618	382	381
380	322	623	624	376	375	627	678	629	371
331	632	633	367	366	365	364	638	669	640
641	642	358	357	345	646	354	353	649	660
651	659	348	347	655	356	344	343	652	650
661	662	668	337	336	335	334	663	639	370
330	672	673	677	326	325	674	628	379	321
320	319	683	684	686	685	617	388	312	311
700	309	308	694	695	606	395	303	302	691

300	202	703	704	296	295	707	798	709	291
211	712	713	287	286	285	284	718	789	720
721	722	278	277	225	726	274	273	729	780
731	269	268	234	735	766	737	263	262	740
260	259	243	744	745	746	757	748	252	251
250	249	753	754	756	755	747	258	242	241
770	239	238	764	765	736	267	233	232	761
771	779	228	227	775	276	224	223	772	730
781	782	788	217	216	215	214	783	719	290
210	792	793	797	206	205	794	708	299	201

801	802	198	197	105	806	194	193	809	900
811	189	188	114	815	886	817	183	182	820
180	179	123	824	825	826	877	828	172	171
170	132	833	834	166	165	837	868	839	161
141	842	843	157	156	155	154	848	859	850
851	852	858	147	146	145	144	853	849	160
140	862	863	867	136	135	864	838	169	131
130	129	873	874	876	875	827	178	122	121
890	119	118	884	885	816	187	113	112	881
891	899	108	107	895	196	104	103	892	810

100	99	3	904	905	906	997	908	92	91
90	12	913	914	86	85	917	988	919	81
21	922	923	77	76	75	74	928	979	930
931	932	68	67	35	936	64	63	939	970
941	59	58	44	945	956	947	53	52	950
960	49	48	954	955	946	57	43	42	951
961	969	38	37	965	66	34	33	962	940
971	972	978	27	26	25	24	973	929	80
20	982	983	987	16	15	984	918	89	11
10	9	993	994	996	995	907	98	2	1

Fig. 31. Last 4 plates of 10^3, made from Fig. 29. (Sum = 5005.)

If we attack 6^3 by the general rule, we find 4 symbols, ×, —, |, ·, and only 3 cells in the rod; the construction is therefore impossible. Suppose we construct an index-cube from the rod [×|—], we shall find it impossible to distribute the remaining symbol [·] in the extended reversion-scheme obtained from this index. The feat, however, is possible if we make (for this case only) a slight change in the meanings of | and —. By the general rule × operates on 4 cells in its own P-plane, where, by the rule of association, the planes are paired thus: $\left|\begin{array}{ll} 1 & \text{with } 6 \\ 2 & \text{“ } 5 \\ 3 & \text{“ } 4 \end{array}\right|$. In interpreting

the meanings of | and —, in this special case, we must make a cyclic change in the right-hand column of this little table. Thus for "|" $\left|\begin{matrix}1 \text{ with } 5\\ 2 \quad " \quad 4\\ 3 \quad " \quad 6\end{matrix}\right|$, and for "—" $\left|\begin{matrix}1 \text{ with } 4\\ 2 \quad " \quad 6\\ 3 \quad " \quad 5\end{matrix}\right|$. This means that a [|], for example, in the second P-plane has its usual meaning in that plane, and also acts on the two cells which would be the associated cells if the 4th plane were to become the 5th, etc. If we extend this scheme, there will be just room to properly distribute the [·]'s in the two parallelopipeds which form the right-hand upper and left-hand lower quarters of the cube, as shown in Fig. 32.

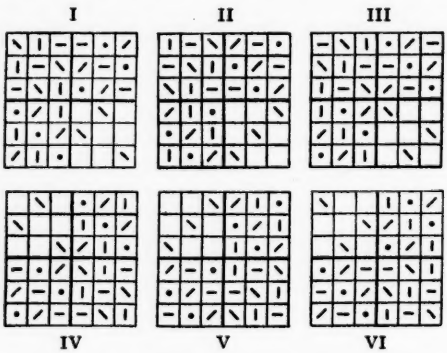


Fig. 32. Extended Reversion-Scheme for 6^3.

This scheme produces the cube shown below, which is magic on its 36 rows, 36 columns, 36 lines, *and on its* 4 *central great diagonals.*

Fig. 32 is the identical scheme discovered by Firth in 1889, and was obtained a few months later than the pseudo-skeleton shown in Fig. 21. A year or two earlier he had discovered the broken reversion for squares of even order, but he never generalized the method, or conceived the idea of an index-cube. The development of the method as here described was worked out by the present writer in 1894.

About the same time Rouse Ball, of Trinity College, Cambridge, independently arrived at the method of reversions for squares (compare the earlier editions of his *Mathematical Recreations,* Macmillan), and in the last edition, 1905, he adopts the idea of an index-square; but he makes no application to cubes or higher dimensions. There is reason to believe, however, that the idea of reversions by means of an index-square was known to Fermat. In his letter to

I

216	32	4	3	185	211
25	11	208	207	8	192
18	203	21	196	200	13
199	197	15	22	194	24
7	206	190	189	29	30
186	2	213	34	35	181

II

67	41	178	177	38	150
48	173	63	154	170	43
168	56	52	51	161	163
162	50	165	58	59	157
169	155	45	64	152	66
37	176	148	147	71	72

III

78	143	105	112	140	73
138	98	82	81	119	133
91	89	130	129	86	126
85	128	124	123	95	96
120	80	135	100	101	115
139	113	75	106	110	108

IV

109	107	111	76	104	144
102	116	117	136	83	97
121	122	94	93	131	90
132	92	88	87	125	127
84	137	99	118	134	79
103	77	142	141	74	114

V

145	146	70	69	179	42
151	65	153	46	62	174
60	158	159	166	53	55
54	167	57	160	164	49
61	47	172	171	44	156
180	68	40	39	149	175

VI

36	182	183	214	5	31
187	188	28	27	209	12
193	23	195	16	20	204
19	17	202	201	14	198
210	26	10	9	191	205
6	215	33	184	212	1

Fig. 33, made from Fig. 32. Sum = 651.

Mersenne of April 1, 1640, (*Œuvres de Fermat*, Vol. II, p. 193), he gives the square of order 6 shown in Fig. 34. This is obtained by applying the index (Fig. 35) to the *bottom* left-hand corner of the natural square written from below upwards, i. e., with the numbers 1 to 6 in the bottom row, 7 to 12 in the row above this, etc. There is nothing surprising in this method of writing the natural square, in fact it is suggested by the conventions of Cartesian geometry, with which Fermat was familiar. There is a

much later similar instance: Cayley, in 1890, dealing with "Latin squares," writes from below upwards, although Euler, in his original Memoire (1782), wrote from above downwards. Another square of order 6, given by Fermat, in the same place, is made from the same index, but is disguised because he uses a "deformed" natural square.

6	32	3	34	35	1
7	11	27	28	8	30
19	14	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

Fig. 34.

−	1	×
1	×	−
×	−	1

Fig. 35.

It is interesting to note that all these reversion magics (unlike those made by Thompson's method), are La Hireian, and also that the La Hireian scheme can be obtained by turning a single outline on itself. To explain this statement we will translate the square in Fig. 12 into the scale

A

55	04	52	03	01	50
40	44	13	12	41	15
25	31	33	32	24	20
30	21	23	22	34	35
10	14	42	43	11	45
05	51	02	53	54	00

B

Fig. 36.

whose radix is 6, first decreasing every number by unity. This last artifice is merely equivalent to using the n^2 consecutive numbers from 0 to n^2—1, instead of from 1 to n^2, and is convenient because it brings the scheme of units and the scheme of 6's digits into uniformity.

If we examine this result as shown in Fig. 36 we

find that the scheme for units can be converted into that for the 6's, by turning the skeleton through 180° about the axis AB; that is to say, a single outline turned upon itself will produce the magic.

I

555	051	003	002	504	550
040	014	543	542	011	515
025	534	032	523	531	020
530	524	022	033	521	035
010	541	513	512	044	045
505	001	552	053	054	500

II

150	104	453	452	101	405
115	444	142	413	441	110
435	131	123	122	424	430
425	121	432	133	134	420
440	414	112	143	411	145
100	451	403	402	154	155

III

205	354	252	303	351	200
345	241	213	212	314	340
230	224	333	332	221	325
220	331	323	322	234	235
315	211	342	243	244	310
350	304	202	253	301	255

300	254	302	203	251	355
245	311	312	343	214	240
320	321	233	232	334	225
335	231	223	222	324	330
215	344	242	313	341	210
250	204	353	352	201	305

400	401	153	152	454	105
410	144	412	113	141	445
135	421	422	433	124	130
125	434	132	423	431	120
140	114	443	442	111	415
455	151	103	102	404	450

055	501	502	553	004	050
510	511	043	042	544	015
520	034	522	023	031	535
030	024	533	532	021	525
545	041	013	012	514	540
005	554	052	503	551	000

Fig. 37.

The same is true of the cube; that is, just as we can obtain a La Hireian scheme for a square by turning a single square outline once upon itself, so a similar scheme for a cube can be obtained by turning a cubic outline

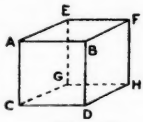


Fig. 38.

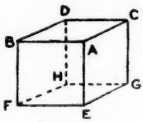


Fig. 39.

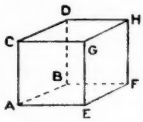


Fig. 40.

twice upon itself. If we reduce all the numbers in Fig. 33 by unity and then "unroll" the cube, we get the La Hireian scheme of Fig. 37 in the scale radix 6.

If now we represent the skeleton of the 6^2's: (left-hand) digits by Fig. 38, and give this cube the "twist" indicated

by Fig. 39, we shall get the skeleton of the 6's (middle) digits, and the turn suggested by Fig. 40 gives that of the units (right-hand) digits. Thus a single outline turned twice upon itself gives the scheme.

We can construct any crude magic octahedroid[9] of

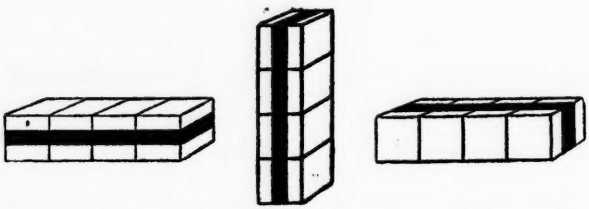


Fig. 41, 1st reversion. Fig. 42, 2d reversion. Fig. 43, 3d reversion.

double-of-even order, by the method of reversions, as shown with 4^4 in Figs. 41 to 44.

The first three reversions will be easily understood from the figures, but the fourth requires some explanation It actually amounts to an interchange between every pair

Fig. 44, 4th reversion.

of numbers in associated cells of the parallelopiped formed by the two central cubical sections. If the reader will use a box or some other "rectangular" solid as a model, and number the 8 corners, he will find that such a change cannot be effected in three-dimensional space by turning the

[9]

Dimensions	Regular Figure	Boundaries
2	*Tetra*gon (or square)	*4* one-dimensional straight lines
3	*Hexa*hedron (cube)	*6* two-dimensional squares
4	*Octa*hedroid	*8* three-dimensional cubes
etc.	etc.	etc.

parallelopiped as a whole, on the same principle that a right hand cannot, by any turn, be converted into a left hand. But such a change can be produced by a single turn in 4-dimensional space; in fact this last reversion is made with regard to an axis in the 4th, or imaginary direction.

1	2	3	4
248	247	246	245
252	251	250	249
13	14	15	16

65	66	67	68
184	183	182	181
188	187	186	185
77	78	79	80

129	130	131	132
120	119	118	117
124	123	122	121
141	142	143	144

193	194	195	196
56	55	54	53
60	59	58	57
205	206	207	208

17	18	19	20
232	231	230	229
236	235	234	233
29	30	31	32

81	82	83	84
168	167	166	165
172	171	170	169
93	94	95	96

145	146	147	148
104	103	102	101
108	107	106	105
157	158	159	160

209	210	211	212
40	39	38	37
44	43	42	41
221	222	223	224

33	34	35	36
216	215	214	213
220	219	218	217
45	46	47	48

97	98	99	100
152	151	150	149
156	155	154	153
109	110	111	112

161	162	163	164
88	87	86	85
92	91	90	89
173	174	175	176

225	226	227	228
24	23	22	21
28	27	26	25
237	238	239	240

49	50	51	52
200	199	198	197
204	203	202	201
61	62	63	64

113	114	115	116
136	135	134	133
140	139	138	137
125	126	127	128

177	178	179	180
72	71	70	69
76	75	74	73
189	190	191	192

241	242	243	244
8	7	6	5
12	11	10	9
253	254	255	256

Fig. 45.

The following four figures (45-48) show each stage of the process, and if the reader will compare them with the results of a like series of reversions made from a different aspect of the natural octahedroid, he will find that the "imaginary" reversion then becomes a real reversion, while

one of the reversions which was real becomes imaginary. Fig. 45 is the natural 4^4 after the first reversion, magic in columns only; Fig. 46 is Fig. 45 after the second reversion, magic in rows and columns; Fig. 47 is Fig. 46 after the third reversion, magic in rows, columns and lines; and

1	254	255	4
248	11	10	245
252	7	6	249
13	242	243	16

65	190	191	68
184	75	74	181
188	71	70	185
77	178	179	80

129	126	127	132
120	139	138	117
124	135	134	121
141	114	115	144

193	62	63	196
56	203	202	53
60	199	198	57
205	50	51	208

17	238	239	20
232	27	26	229
236	23	22	233
29	226	227	32

81	174	175	84
168	91	90	165
172	87	86	169
93	162	163	96

145	110	111	148
104	155	154	101
108	151	150	105
157	98	99	160

209	46	47	212
40	219	218	37
44	215	214	41
221	34	35	224

33	222	223	36
216	43	42	213
220	39	38	217
45	210	211	48

97	158	159	100
152	107	106	149
156	103	102	153
109	146	147	112

161	94	95	164
88	171	170	85
92	167	166	89
173	82	83	176

225	30	31	228
24	235	234	21
28	231	230	25
237	18	19	240

49	206	207	52
200	59	58	197
204	55	54	201
61	194	195	64

113	142	143	116
136	123	122	133
140	119	118	137
125	130	131	128

177	78	79	180
72	187	186	69
76	183	182	73
189	66	67	192

241	14	15	244
8	251	250	5
12	247	246	9
253	2	3	256

Fig. 46.

Fig. 48 is Fig. 47 after the fourth reversion, magic in rows, columns, lines and *i*'s, = crude magic 4^4. The symbol *i* denotes series of cells parallel to the imaginary edge.

Fig. 48 is magic on its 64 rows, 64 columns, 64 lines, and 64 *i*'s, and on its 8 central hyperdiagonals. Through-

out the above operations the columns of squares have been taken as forming the four cells of the P_1-aspect;[10] the rows of squares taken to form cubes, of course, show the P_2-aspect.

1	254	255	4
248	11	10	245
252	7	6	249
13	242	243	16

65	190	191	68
184	75	74	181
188	71	70	185
77	178	179	80

129	126	127	132
120	139	138	117
124	135	134	121
141	114	115	144

193	62	63	196
56	203	202	53
60	199	198	57
205	50	51	208

224	35	34	221
41	214	215	44
37	218	219	40
212	47	46	209

160	99	98	157
105	150	151	108
101	154	155	104
148	111	110	145

96	163	162	93
169	86	87	172
165	90	91	168
84	175	174	81

32	227	226	29
233	22	23	236
229	26	27	232
20	239	238	17

240	19	18	237
25	230	231	28
21	234	235	24
228	31	30	225

176	83	82	173
89	166	167	92
85	170	171	88
164	95	94	161

112	147	146	109
153	102	103	156
149	106	107	152
100	159	158	97

48	211	210	45
217	38	39	220
213	42	43	216
36	223	222	33

49	206	207	52
200	59	58	197
204	55	54	201
61	194	195	64

113	142	143	116
136	123	122	133
140	119	118	137
125	130	131	128

177	78	79	180
72	187	186	69
76	183	182	73
189	66	67	192

241	14	15	244
8	251	250	5
12	247	246	9
253	2	3	256

Fig. 47.

This construction has been introduced merely to accentuate the analogy between magics of various dimensions; we might have obtained the magic 4^4 much more

[10] Since the 4th dimension is the square of the second, two aspects of the octahedroid are shown in the presentation plane. The 3d and 4th aspects are in H-planes and V-planes. Since there are two P-plane aspects it might appear that each would produce a different H-plane and V-plane aspect; but this is a delusion.

rapidly by a method analogous to that used for 4^3 (Fig. 26). We have simply to interchange each number in the natural octahedroid occupying a cell marked [×] in Fig. 49, with its complementary number lying in the associated cell

1	254	255	4
248	11	10	245
252	7	6	249
13	242	243	16

192	67	66	189
73	182	183	76
69	186	187	72
180	79	78	177

128	131	130	125
137	118	119	140
133	122	123	136
116	143	142	113

193	62	63	196
56	203	202	53
60	199	198	57
205	50	51	208

224	35	34	221
41	214	215	44
37	218	219	40
212	47	46	209

97	158	159	100
152	107	106	149
156	103	102	153
109	146	147	112

161	94	95	164
88	171	170	85
92	167	166	89
173	82	83	176

32	227	226	29
233	22	23	236
229	26	27	232
20	239	238	17

240	19	18	237
25	230	231	28
21	234	235	24
228	31	30	225

81	174	175	84
168	91	90	165
172	87	86	169
93	162	163	96

145	110	111	148
104	155	154	101
108	151	150	105
157	98	99	160

48	211	210	45
217	38	39	220
213	42	43	216
36	223	222	33

49	206	207	52
200	59	58	197
204	55	54	201
61	194	195	64

144	115	114	141
121	134	135	124
117	138	139	120
132	127	126	129

80	179	178	77
185	70	71	188
181	74	75	184
68	191	190	65

241	14	15	244
8	251	250	5
12	247	246	9
253	2	3	256

Fig. 48.

of the associated cube. Fig. 49 is the extended skew-reversion scheme from the index-rod [□×].

All magic octahedroids of double-of-odd order $>10^4$ can be constructed by the index-rod, for just as we construct an index-square from the rod, and an index-cube from the square, so we can construct an index-octrahedroid

from the cube. The magics 6^4 and 10^4 have not the capacity for construction by the general rule, but they may be

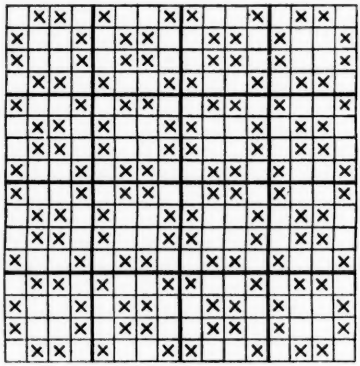


Fig. 49. Skew Reversion for 4^4.

obtained by scattering the symbols over the whole figure as we did with 6^3.

C. PLANCK.

HAYWARD'S HEATH, ENGLAND.

TWO STUDIES IN SUGGESTION.

THE BOXERS.

ALL the world knows how the North of China was convulsed in the year 1900 by a wave of patriotic feeling stimulated by certain enthusiasts named by foreigners "Boxers." It is not quite so well known that this enthusiasm was propagated by recognized methods of psychical excitement.

This society, known as the *I ho ch'üan* or "Public Harmony Fists," arose in Shantung province, and, by the connivance of certain local officials whose national feelings outran their prudence, expanded and spread throughout that province and into the adjoining one of Chih-Li. In the course of the summer of 1900 all the provinces north of the Yellow River were permeated, the matter coming to a climax in the famous siege of the Peking legations.

All narrators agree that certain rites were performed by the propagators of the movement, which came to receive the vague title of "Boxer drill."

The following quotations will indicate the general nature of this process:

A. "They were not successful in getting the people to take it up at first, so they began with boys ten to twelve years of age.... After a few days it grew very rapidly. The drill, if it may be called so, consists in the boy repeating four short lines of some mystic words, and bowing to the south and falling backwards, when he goes into a trance,

remaining lying on his back for an indefinite time, when he rises and is endowed with wonderful strength, boys of twelve being as strong as men. They brandish swords and spears, not seeming to try to be skilful in handling them, but merely to show strength and place themselves under the protection of their symbols. They claim to be invulnerable."—Rev. C. W. Price of Fen-Chou-Fu, Shansi, in *With Fire and Sword in Shan-si* (Diary, June 1, 1900).

B. "Drill consisted in incense before a tablet. . . .and then working themselves by gymnastics, etc., into a state when they were no longer masters of themselves, but became unconscious. After remaining in this state for some time they would rise, declaring themselves possessed by the spirit of one of the heroes of antiquity. In this state they could perform great feats, but the chief mark was that they were invulnerable. Swords did not hurt, and they knocked their heads till great bumps appeared, but never felt it."—Slightly abridged from Mrs. A. H. Mateer, *Siege Days,* New York, Redell.

According to the Rev. G. T. Candlin (author of *Chinese Fiction,* Chicago, Open Court Pub. Co.) who was in Tung Shan during the outbreak, the "four mystic lines" were as follows, and were accompanied by certain postures (bowing in the Chinese ceremonial style of prostrating and beating the head on the ground):

T'ien ta, T'ien chiu k'ai
Ti ta, Ti chiu k'ai
Yao hsüeh I ho ch'üan
Huan tê Shih Fu lai.

"Beat the heaven, the heaven will open;
Beat earth, and earth will open;
Desire to learn the public-harmony-force[1]
Also get the masters to come."

He has also expressed an opinion that Buddhist and Taoist priests were connected in some way with the move-

[1] *Ch'üan* is "fist" but has in this case the sense of the power of the fist.

ment and employed hypnotic methods. In this connection it is interesting to note that Putnam Weale in his famous book *Indiscreet Letters from Peking* speaks of a temple which had been specially frequented by Boxers, and that native Christians had been murdered there, presumably in some sense as sacrifices. He also mentions the large part played by boys in the movement.

If we survey the whole of the information available (of which the above is but a representative selection) it is evident that

1. Ceremonial rites including prostrations and chants formed the initial feature of the process and were prolonged until the cerebral consciousness became dormant;

2. A period of trance supervened;

3. The trance was followed by a period of great excitement in which excessive muscular energy and anesthesia were shown;

4. Boys were more subject to the influence than men, but once started it was very contagious;

5. The dominant idea was to expel the foreigner, and this was readily acceptable to the people at the time on account of public events. This was shown in the motto

Pao ch'ing, mieh yang,

"Guard the Ch'ing Dynasty, destroy the foreigner."

The Chinese are peculiarly subject to the suggestive value of epigrammatic sentences like this, and in this case we have not far to look for the master-thought.

The belief in possession by spirits is of course not peculiar to them, but an example of it in China is given in the Rev. MacGowan's book on *Side Lights on Chinese Life,* quoted in my article on "Chinese Philosophy and Magic" in the *Journal of the Royal Society of Arts,* April 21, 1911. The Confucian philosophy as expounded by Chu Hsi implies that the vital spirit in men is one and

the same with that of their ancestors, so that it is not difficult for them to conceive that the peculiar individuality of an heroic ancestor may well up in the soul of his descendant. Such an idea forms a simple (and to them, rational) explanation of the enthusiasm and modification of personality which immediately succeeded the trance.

The words *Shih-Fu,* "master," may be taken as singular, plural or general, just as one thinks fit. Probably the intellects of the I Ho Ch'üan would instruct their followers specially as to the particular incarnation with which they happened to be favored.

The word *ta,* "to beat," is of very great idiomatic power in Mandarin speech, and must not necessarily be taken in its literal sense. It can mean "to appeal to," or "to serve," and undoubtedly is to be so taken.

Heaven and earth are of course the great Chinese polarities, the reservoirs of positive and negative energy.

The general sense of the chants is then that heavenly and earthly powers will respond if called upon, so that one should desire the patriotic vigor and call upon the dead to enthuse one.

The phenomenon of anesthesia (incorrectly regarded as invulnerability) is of course a usual concomitant of hystero-epilepsy. The dauntless frenzy of the Mahdi's followers undoubtedly sprang from the same conviction of personal safety, their master having assured them that neither sword nor bullet could harm them.

The success of the influence with boys indicates the hindrances which the auto-suggestions of reason placed in the way of the submission of adults. Boys have universally been employed as "mediums" in the East.[2]

An interesting point in the whole question is whether it was incepted by intellects who understand more or less well the laws of psychology, or merely arose from the nat-

[2] See Lane's *Modern Egyptians.*

ural aggregation of anti-foreign influences. It will perhaps be useful to consider what are the Chinese notions as to psychology.

Primitive Psychology in China.

The only character in the ancient Chinese hieroglyphics which takes a permanent place in psychological ideas is *hsin,* "the heart." Egyptian and Semitic literature show the same feature. In all three languages other symbols are used for external quasi-psychical phenomena, but the individual's own feelings and thoughts are almost all expressed in terms of the "heart." In other words, the heart was regarded as the seat of the intellect and emotions, presumably because the emotions when of a violent character affect the "sympathetic" or ganglionic nervous system and the heart shows the disturbance most strongly. As example we cite the following compound characters:
The term "virtue" consists of a radical meaning "to walk" combined with "straight" and "heart."
The character "like" above "heart" means "reciprocity."
The character "slave" above "heart" means "anger."
The character "receive" above "heart" means "love."
The character "inferior" above "heart" means "hate."
The character "scholar" above "heart" means "will."
The character "mutual" above "heart" means "think."
The character "middle" above "heart" means "sincerity."

Dual Consciousness in Chinese Psychology.

The distinction between the central energies of the cerebro-spinal nervous system and those of the ganglionic (sympathetic) system has only recently been made out (See Hudson's *Law of Psychic Phenomena*) and is clearly adumbrated in the scholastic *animus* and *anima* and the Chinese *hun* and *p'o.* These are the personalized forms of the psychic quantities *shen* and *kuei.* The energies are re-

spectively termed *ch'i* and *ching,* and are regarded as special forms of the positive (*yang*) and negative (*yin*) polarities of energy. The *hun* is supposed to wander at times during life and after death, while the *p'o* controls the animal functions and only persists in a shadowy form after death. Stimulated, the *hun* manifests as *chih* the will, while the *p'o* is the seat of emotion, *ch'ing*.

The ideas outlined above are almost all that can be gleaned from the ordinarily accessible native works. The practice of meditation in Buddhist and Taoist monasteries is undoubtedly based on careful observations of the results of "religious exercises." The Rev. Timothy Richard of Shanghai has translated a book which he calls the "Guide to Buddhahood," *Hsüan Fo p'u* (literally "The Record of the Selection of the Buddha").

This is a graduated statement of the development of the soul on ecstatic lines and reminds one of St. Teresa's *Castillo Interior.* Commencing with introspection of morals, it passes to contemplation of virtue and then through a whole series of meditations on mythological concepts, which will culminate in Nirvana. The analogy with the stages of apotheosis described by Plotinus and the Sufis is obvious.

An acquaintance with such mental conditions (probably accompanied by strange phenomena in various cases due to the nervous idiosyncrasies of the individual) would be quite sufficient to provide a working hypothesis for such a movement as that of the I-ho-ch'üan. By those who care for the terminology it may be legitimately called "black magic," although it amounts to very little more than the control exerted by religious fanatics generally on those of their disciples who have been "worked up" to the point of hysteria. There is this difference, however, that in the East the moving spirits generally know to some extent what they are doing, whereas in the West this knowledge

is only possessed by those who have little or no occasion to employ it.

In conclusion the writer would point out that the normal Chinese mind is very acute, but conservative and lacking initiative. When excited however beyond a certain point, it exhibits a wild frenzy which is utterly reckless of consequences. These characteristics of course are not peculiar to the people but seem, at least to the writer, to be more marked than in the European. Speaking broadly, the European in China behaves as if he had little or no self-control in small matters, whereas in important things he generally becomes cool. With the Chinese it is the reverse, perfect nonchalance in ordinary affairs but imperfect balance in large ones. The writer does not of course suggest this is universally true of individuals.

THE MAGICAL USE OF BLOOD.

In the highest and lowest of ceremonial religions, and almost universally in connection with magic, we find references to the potency of blood.

The standard methods of ancestor-worship[3] include a bloody sacrifice to the manes, and an anointing with blood of the eidolon which represents the spirit. Primitively the blood is placed in the mouth of the figure. Almost in all cases it is conceived as providing vitality to the ghost. The invocation of the ghosts in the Odyssey is a typical case. The Pentateuch says "the blood is the life," and to this day the Jews abstain from meat which is not *kosher,* i. e., deprived of blood.

In China there are similar notions. Thus under the character *hsüeh,* "blood," in Giles's Dictionary the following phrase occurs:

Jen hsüeh chih wei yeh huo yeh, "Man's blood causes strange fire."

[3] See Grant Allen's *Evolution of the Idea of God.*

This emanation from blood is also termed *kuei huo,* "ghostly fire."

Again in the medieval books on magic we find that

1. Numerous prescriptions and charms require blood, and even bloody sacrifices are necessary in some cases;

2. Books professing to teach only pure theurgy recommend the practitioner to avoid the use of blood.

The aversion for blood also appears in the practices of bloodless execution employed by the Turks and the Inquisition.

The atoning power of blood is referred to in the seventeenth chapter of Leviticus, and developed in Christianity into the eucharistic sacrifice. It is also fairly clearly recognized in all bloody rites performed in the service of spirits.

Other references can be drawn from numerous sources. The marvelous blood-stains which remain on hero's sword and in haunted house; the practice of signing important acts (such as pacts with the devil!) in blood; the impurity of blood when on the person; all illustrate the general conception of its extraordinary properties.

The persistence and generality of such ideas point to some underlying psychical fact. At first sight the common experience of nausea or fainting at the sight of blood might be regarded as the fundamental cause, but a little consideration will show that this is either one of the effects of the cause we seek or a vestigial retro-reminiscence of the beliefs on the subject which dominated our forefathers.

To the writer it appears that the mere continuous juxtaposition of blood with pain and death in common experience, extending through untold generations, is quite sufficient to account for the effects and beliefs which have been referred to, acting in accordance with the laws of psychic change. In minds which have not been trained to oppose the quasi-mechanical suggestions of revived memories by

specially developed associations (religious or scientific), the percept of blood will immediately call up memories of pain and death. These again will be followed by memories of incipient insensibility and fear, which will tend to be realized again in the organism by a partial paralysis of the motor centers etc., i. e., the organism will reproduce as far as possible the state remembered.

These changes, proceeding from a cause not immediately apparent to sense, are naturally ascribed to an external source, more particularly in view of the fact that another person (such as a wizard) can by insistent suggestion (with or without hypnosis) set the train of transformation in motion.

Blood has a quite perceptible smell (the extraordinary sensitiveness of carnivorous animals and insects needs only to be mentioned in support of this fact) and a perception of this is sufficient to provide a basis for the belief in peculiar sanguinary emanations. Add to this the obvious connection between blood and vitality, and we have a complete nexus of percepts which will suggest all the magical ideas mentioned, and by the encouragement of such suggestions will tend to realize the psychical counterparts of such magical causes.

Some modifications in this statement may be conceded to those who under the vague name of occultists contend that a whole series of supernormal laws continuously operates on human affairs. Such will say that all the properties attributed to blood in universal spiritualistic belief are real, that spirits (shells) can absorb sanguinary emanations and thereby vitalize themselves, etc. To them it may be said that using the word "spirit" as equivalent to "idea" the difference is merely a matter of terminology.

HERBERT CHATLEY.

T'ANG SHAN, NORTH CHINA, Oct. 1911.

AUTOMATISM.

IN approaching a subject of such an uncertain nature, of such wide bearing and interest to humanity, and resting on the much disputed border of the unknown, it is only with the greatest regard for fact and approved hypotheses, and the utmost caution in reasoning that I have felt myself at all capable of developing it to any conclusion. The nature of the subject forbids any actual proof by our present facilities and in no place would I wish to assume my own infallibility. While the metaphysics of the question is, at present, of no practical use or bearing, yet a knowledge of the government of our actions and a conception of what this government and its rules should be, I may state to be the thing of highest utility and interest to us. According to Mill, "no belief which is contrary to the truth can be really useful," and so, at least, there is some excuse, aside from complete treatment of the subject, for developing its metaphysical side before proceeding to that of more immediate utility—the educational and moral phases. Many treatises and good have been written upon this subject, and many strong arguments *pro* and *con* adduced, but there is always a last word to be said, and the best inferences and reasons have been put to shame as the truth has slowly come to light.

No man is so presumptuous as to assert that he recognizes all causes which tend toward the production of any phenomenon, but a faith that they exist and are discoverable, is what has led to the present glory and brilliance of

science. Man wonders and is curious now even as he was in the dim ages, but he has learned one lesson,—to investigate for natural causes instead of "explaining away" his ignorance by the creation of supernatural powers; and the answers which he gives to the questions of the universe to-day are not mere placebos to console his passion for an answer and to feed his emotions, but passion has been supplanted by a higher and more lasting emotion; namely, the desire for the satisfaction of reason with positive and logically deduced knowledge; and nothing more and nothing less will suffice.

In order to conform to this inner desire and all that is implied with it, it is not necessary to exclude all belief and remain purely agnostic, but to have that belief bounded and governed by the known facts of science and its articles determined by the most plausible inferences adducible therefrom. All men, no matter in what age or circumstances, have with the greatest legitimacy constructed a cosmos and not a chaos as their picture of the nature of things. For do they not see around them at all times direct evidence of law and order in the workings of all material forces? And the least of confirmation is a pillar to belief.

To develop here whatever system of belief might be entertained with the sanction of facts would hardly be within the confines of my subject, but suffice it to say that I agree with Spinoza who says that "an appeal to the interference of a soul (or unknown spiritual force) in order to explain a corporeal state, is an admission that we do not know its cause." I can in no way sympathize with the inert mind of the Orient which, too drunk with sun and plenty, must depend upon the spirit to fill the vacancy in its knowledge,—a spirit about which it has even less of an idea than of the material phenomenon itself. In the absence of knowledge we are only justified by an inference which we believe to be in the direction pointed out by facts.

Now the material and its actions are the only facts with which we are acquainted. Science has classified these facts of experience and induced laws therefrom and in every case the fact has been of a material and causative nature. It therefore becomes our first duty to attempt the reduction of all phenomena to a physical, substantive basis and not, when we have no conception of the cause, to say that its nature is "spiritual," but courageously to assert our ignorance concerning it and work with the faith that it may be reduced to a natural, materially caused phenomenon. Never *ignorabimus*! I shall preserve this rule, and work with this end in view in all that follows.

It may satisfy some to ease their desire for rationality with the following statement of Haeckel, but, however true, it does not make a direct argument against the reason of the indeterminist, which first of all must be shown fallacious before our own can trust the evidence. Professor Haeckel says, "As to the question of free-will which has kept the world busy for two thousand years, and which has produced so many books that encumber our libraries and accumulate dust therein,—this question also is no more than a memory. Of what value are vague suggestions based upon sentiment, in comparison with scientific deductions? The will indeed is not an inert force. It is a power of automatic and conscious reaction which is regulative and actively influential. But the inclinations that are inseparable from life itself explain this attribute, and as to the mode of action inherent in the will we only consider it free because, following the abstract and dualistic method of metaphysicians, we isolate this faculty from the conditions which determine it. We have not, first of all, to consider the will separately, and then examine the circumstances wherein it acts. The will as given is burdened with a thousand determinations which heredity has settled upon it. And each of its resolutions is an adaptation of its

pre-existing inclination to actual circumstances. The strongest motive prevails mechanically by virtue of the laws which govern the statics of emotion. If then the merely abstract and verbal will appears free, the concrete will is determined like everything else in the universe."

To say this in the face of the overwhelming number of scientific and unscientific indeterminists is not enough, and it is the object of this essay to adduce such reasons as will lead to the establishment of these statements as facts. In doing so, let me say that I do not consider it inconsistent to accept and reason from the tried theories of science which have stood the test of time and criticism.

We know that during that comparatively simple condition of the earth, before the Laurentian age and the primordial deposits, a simple organic unit was produced. Bernard, who has made the cell his life-study, has reduced the cell, which had been formerly considered the unit of structure, to what he terms the "chromidial unit," a more elementary organic structure, having as definite a morphological significance in its own way as the cell. It can be claimed therefore that some such unit produced all the pre-cellular organisms which built up, among other less successful organisms, the famous cell with which biologists usually start their record of life. Not only was the cell a highly efficient organism in itself, as is shown by the fact that so many unicellular organisms exist to-day, but it had the power of multiplying indefinitely and forming colonies, which colonies have become organisms specialized to numberless more and ever more complicated environments. For the specialization of a large colony of cells as a whole must necessarily be able to reach a level of complexity higher than that to which any single cell could possibly attain. So thus life was raised from one level of complexity to a higher one, and it is by comparatively little reasoning that we reach the age of man.

Now in the simple stage of the earth's history and even later in the postcellular age, it is acknowledged that all phenomena obeyed explicitly the omnipotent, omnipresent law of cause-effect. The actions of the ameba are nothing but the simplest of reflexes from external stimuli and this same action is admitted to continue up to the lower vertebrates. All those who have expounded the doctrine of free will have, therefore, consciously or unconsciously, stated that at some unknown instant of time in the slow, gradual evolution of organic life, and also in the growth of the embryo or early life of the infant, the animal has ceased to act according to the natural laws of its previous action and a force has crept into a universe which embraces all space, which is able to produce material phenomena on its own account and aside from the law that all motion possesses a cause of which it is the direct effect. Is it not absurd to hold that the action of a few of the higher animals are not caused and so proceed by "their own virtue?" Is this not exactly how primitive man "explained away" any phenomenon of the cause of which he was ignorant? How unreasonable it is when we realize the complex nature of the subject of our study and the complex environment upon which he must react, to infer that his action is not a more complex one working by the same rules as his simpler action did in past ages at the time of his humble origin. It is a case of realizing that a million phenomena whose cause is known to reside in a certain law, surround one phenomenon,—that of the action of the higher animals, the complexity of which has baffled our investigation, and therefore that we do not assign this one to law, but label it a causeless phenomenon the action of which is based upon the "virtue of the will." It is the insignia and confession of the lack of knowledge and the lack of inductive reasoning power of a great number of our professed scientific thinkers. They should ob-

serve their rule, namely, that if the law applies in a thousand cases the probability is a thousand to one that it will apply in the thousand and first case.

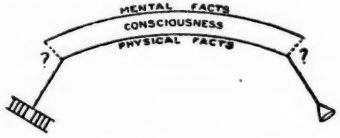
There is also another serious difficulty which presents itself to the exponents of free will, and as yet none have replied successfully to Professor Clifford who I believe was the first to discover it. In extract it is this: the will, in being pure and uninfluenced in its choice or production of a material phenomenon, and therefore free, as they say, must, in not being governed according to cause-effect, influence matter through the immaterial; and aside from the fact that the existence of the immaterial is inconceivable, otherwise than that matter should be governed by anything but surrounding matter is also inconceivable, and both are therefore highly improbable. The conclusion therefore is inevitable that the will is a physical manifestation and governed by the laws of physics.

No real boundary exists between the unconscious involuntary actions of instinct born in us or of habits formed, and the subconscious "quasi-voluntary" action of brushing the dust off one's sleeve during a conversation, or between the subconscious and more complex reactions in full consciousness. It is a known fact that when the higher forms of memory appear in animal life, a fuller and more complete consciousness exists. And this is necessarily the case, for in order to obtain the more complex reactions of the higher animals, it is necessary that a greater memory of the results of actions be had and so a fuller consciousness for the revolving of the many memories to obtain the most favorable idea of the would-be consequence and so its enactment. For the most favorable memory or idea of the consequences of actions determines our choice, on account of the self-instinct necessitated by the law of natural selection and whatever social education we have had.

Most indeterminists, believing that *ab extra* the mental

and physical processes go along on two parallel platforms —the mental activity opposite the corresponding physical activity—are confronted by this argument: Since we are reasoning beings, there is a chain of mental facts between the incoming and a motor action, and so there is a complete chain of physical facts sufficient to produce the action; for before and along with the mental act of willing there is a parallel brain action which is caused and which causes the motor action. There is then no need for the parallel mental process theory, for by its parallelistic nature it destroys our incapacity for accounting for all phenomena physically, which incapacity caused its creation to "explain away" certain of the higher animal actions. The word mental should signify only in consciousness.

Again, how can pure abstract "will" influence material action? Allowing that not only to us but in abstract that "mental" processes intervene between the sensation and motor action, how is one to get across from the physical to the mental platform and then back onto the physical again? This detour, made by metaphysicians on account of ignorance, leads me to doubt the existence of the "mental," immaterial platform. I fail to see the relation between will and motion by which one can cause the other, unless "will" and "mind" are inherent in it, i. e., a manifestation of molecular or molar motion and therefore governed according to cause-effect and not free. The following diagram will illustrate the point:



If it be asserted that the psychical is inherent in inorganic nature, I have nothing to say, for molecular, atomic, and ionic structure is too little known; but I believe

that it is inherent only in the sense that material composition is such that it could produce (by combinations and processes unknown) conscious life. It has been said that consciousness as a form of motion is inconceivable, but admitting its truth I do not consider it a valid argument against materialism, for what kind of an *idea* of consciousness can we have when consciousness can only be the subject and never the object, as we are contained in it?

So in conclusion on the physical facts of the case, the argument may be summed up in these words: In our development from the first transitional organic form to the cell and on through the gastraedic and invertebrate age our condition has resembled that of the monera, amoeba, platode, and up to the lower vertebrates, whose action is so simple that it is readily admitted to be mechanical. But when we come to higher vertebrates and promammals, which we resembled at a more recent period, their constitution and action has become so complex that we must abandon consistency and say: because we see no cause of their actions is there none? No, reason forbids. Upon the fertilization of the ovum and the formation of the stem-cell the life of a human individual begins. This is a mechanical process as is the development of the embryo; the early life of the infant is a combination of instinct and reflex action—purely mechanical. But after the plastic brain substance of the infant has received and held many impressions from the outside world he is equipped for a more complex reaction against it, and since many professed scientists neither realize what his memories are nor see how the most favorable one coupled with the self-preservation instinct sets forth his action, so they assign the action to his pure will to do it and nothing else. Let us use reason in this case. If the action were considered dynamically and an investigation made of its exact molecular cause and its force it would be an operation among those

physiological infinitesimals which present calculation must neglect but to which faith must grant an existence. The removal of the cerebral hemisphere reduces all action to a pure and simple automatic nature and no one has had the opportunity, knowledge or facilities to watch and trace the origin of the so-called voluntary actions in the mysterious mazes of the frontal brain. It remains for us but to wait until methods are so perfected and until men, realizing that knowledge is power, educate themselves unhesitatingly to investigate with a view towards their high aim, upon the highest form of living subject obtainable, for a confirmation of those inferences we have deemed reasonable.

"The higher we ascend in the vertebrate series toward man," says Dr. Carpenter, "the more evident does it become that the ordinary course of action is determined rather by the direction given through the cerebrum to the workings of the automatic mechanism than by its (the cerebrum's) own unconscious action." In other words, by reason rather than by instinct. And in man we find that everything is to be learned by experience, save what is imperatively required for the maintenance of life—such as the rhythmical contractions of the heart, the peristaltic movements of the alimentary canal, the acts of swallowing and respiration and the like. It has already been mentioned that memory is the great prerequisite for all "voluntary" action, and it it is also known that the actions of the human embryo are not of that sort until

> "Nature whose heedless might
> Casts like some shipwrecked sailor, the poor babe,
> Naked and bleating on the shores of light."

From that instant the memory is in process of formation, the conscious personality begins, habit adds to the rôle of the involuntary centers, which previously possessed only instinct, and the infant can thus react more perfectly upon complex conditions and exert less effort in the performance

of simple and necessarily repeated actions, for their performance has become habitual and subconscious. Thus he is able to direct his higher activities to the more difficult phases of his being—to this end has the law of natural selection, joined with variation, ever worked in the mental field. This is the pregnant fact upon which I shall build my argument from the mental side of the question.

As shown under hypnosis, an impression of every experience, of the sight of every performance of others, of the result of every action, is indelibly recorded in the brain, whether it ever be brought into consciousness or not. We therefore have for our use the knowledge of the result of a thousand actions, whether it be of the tongue or of the hand. Now we also possess from heredity the overwhelming instinct of self-preservation and its brother, the desire for what is productive of the greatest happiness to us. The following mental process is easily discernible by introspection: a condition arises in the environment necessitating a reaction; the memory arises of certain results upon the individual of an action of his own or of some one else; if it be a favorable result his interest in the possible action is aroused and his attention is then directed toward it; the same occurs (from association of ideas in the memory) to four or five (taking an extreme case of indecision) ideas of possible action; the attention is directed then from one to the other and a comparison of them is made according to the individual's belief in the probable nature of their results; one appears more favorable to his happiness and welfare than the others, whereupon it is acted out. Thus truly considering the necessity of memory, Plato has reason to name it a great and mighty goddess. If it were not for this cause-effect mental process, I would fear greatly for the happiness and interests of the individual, if there could be individual life without it. One of the potent factors in causing such a strenuous advocacy

of free-will is the pride and vanity of man in himself and his powers. But how often has that pride been humbled and how often must it be in the future when such facts as his low origin or his unlikeness to the image of God are forced upon his realization!

The power of suggestion and association of idea with idea, such as I experience as I sit here writing, must also be thoroughly recognized and considered before any validity, let alone prestige, can be given to the statements of an indeterminist. The previous paragraph has shown the method of the objective or higher faculties in arriving at a conclusion for action, but the memory, or what has been termed the unconscious, subjective mind is always amenable to suggestion and will catch the objective faculties off their guard if possible. A friend related an excellent example of this some days ago: A young man who had determined to stop drinking was invited to step into a saloon and have a glass. He was prepared for this and the suggestion brought up the reply no. A few days later an old school friend met him and said, "Let's go in and sit down and talk over old times." He went in and it is unnecessary to say, succumbed. Taking up the association of memories or ideas, let me ask the free-will exponents if the "chance" were at all probable, of my turning ten minutes ago to the beginning of this paragraph and writing the word *the* with which to start this paragraph, the idea of what I have just written springing spontaneously from my brain? I also ask them to exercise their powers of introspection until they have gained proficiency enough to trace back *why* they did this or thought that the moment before, winding the string (of cause-effect) as they go along and reach, say, their experience an hour ago. My opponents paradoxically admit that they are not reasoning men, for they say they do just as their "free will" pleases and, although moral men, are not governed by duty, re-

sponsibility, or fear of consequences. Freedom consists of a recognition of facts and a self-government according to them; bondage, of a struggle against them. I have already determined what pleases us,—namely that of which the consequences are productive of our happiness and well-being. And the proud "free-willers," I believe, have some hedonists in their ranks who will acknowledge what pleases them, so their acts being governed by that, they prove themselves traitors to the cause. Many people may become indeterminists and reach that abnormal state of mind in which they can trust themselves to a universe where law and lawlessness interchange indiscriminately, but I confess myself unable to reach that Nirvana.

Our own immediate mental experience, therefore, has shown that we are no exception to the rule (in that we realize the mental antecedent—the why of our purpose) and they are as worthy of confidence, according to Dr. Carpenter, as are "deductions drawn from phenomena outside ourselves, which we can only rightfully interpret on the basis afforded by those very experiences, the test of the validity of such interpretation being furnished by their conformity to our other immediate experiences." It is well known that the hemisphereless frog or pigeon acts automatically when any thing directly stimulating is administered, but remains perfectly passive until then. The hemispheres, therefore, are the seats of higher consciousness wherein a more complex reaction is aroused from *more distant* and *delicate* stimuli from without—after the formation of the memory within—but not less automatic action. Our consciousness of effort arises from the many and intricate processes of conscious reasoning, judgment, etc., before arriving at a decision or choice, and is accompanied by the feeling of effort arising from muscular movement. It has been often urged that, since neurosis can give rise to psychosis, it is surely quite accordant with the

fundamental principle of interaction to affirm that conversely, psychosis can give rise to neurosis, just as the electricity generated in a voltaic battery by chemical change can itself produce chemical change. I quite agree—the psychosis being neurosis consciously felt. The neurosis afferently causes psychosis, i. e., causes will; the psychosis efferently (in regard to the ego) causes neurosis and bodily motion. He simply affirms the chain of cause-effect and the law of the conservation of energy.

In fact, unless combination of memories were used to determine our actions and memory be but a rudiment, or else that memory is used for that purpose now, I can see no object past or present toward which it would be of utility. Darwin and his followers have shown that an animal possesses a function because it was either of use to its ancestors or to itself. Therefore since memory would be useless unless it helped and guided our actions we must concede that it does; and we can also conclude that where animals acted in accordance with a more perfect memory (arising from variation) their actions were more in accordance with the requirements of nature and they more fit to live. Thus natural selection has produced this, as well as all other necessary functions. Who would attempt an explanation of the molecular causes of those imaginary actions in dreams? Memory is involved here but the channels through which we come to those imaginations are so subconscious as to baffle all introspection; yet there is no manifestation of will in them and it is comparatively easy to see the by-cause of conscious volitions.

Under hypnotism the will or judgment is unconscious. The man is under the complete control of the present suggestion. Now we see that it is not a very beneficial reaction when no distinction can be made between the false and true, right and wrong, etc. And thus natural selection gave rise to the will—judgment (comparison of memories)

coupled with action. The comparison was and is necessary for existence by a high reaction even as the hemispheres were and are necessary to a reaction from more distant and delicate stimuli. The hemisphereless frog and the hypnotized man are admitted automatons but when there were neither of these conditions and the reaction was complex—from revolving of memories and comparison of them, and from distant stimuli they were thought uncaused as we had no knowledge or perception of them. Now that we see the *why*, we realize that automatic nature in the absence of hypnoses or presence of the hemispheres as well as in the opposite conditions.

On the freedom of choice this is the sole reply which I find from the indeterministic pen. "And yet on the deterministic doctrine, if I am attracted by the temptation of an immediate but immoral pleasure, and am deterred from it either by a sense of duty or by the fear of the remote consequences of the sin, I have no more 'choice' as to the course I shall take than has the piece of iron that is attracted in opposite directions by two unequal equidistant magnets. Now my contention is not merely that I have a choice, but that the very existence of an idea that can be derived from no other source than human experience, confirms that effect." I believe Dr. Carpenter perfectly justified in making this statement. As to the person it is a choice (at the moment he does not figure out all reasons or causes, they being subconscious), but the "choice," not to us but abstractly, is determined and nonexistent. The fact that all experience shows that motives which may exert a preponderating influence at one moment, are comparatively powerless at another, and that, on the other hand, motives whose influence at one moment is scarcely felt, may come to acquire a force that makes them far outweigh those which at first overbalanced them, shows that, although we do not know what is really the

best decision, if we can be made to believe that a certain one is (by any means whatever) better, that is the one which the self-instinct, or whatever social education we have had, embodies with the proper action. Indeterminism confesses its inability to trace anything behind the will or existing before it which is in any way connected with it; determinism confesses that it sees and also consciously experiences (and what our consciousness tells us is the surest reality to us) a phenomenon existing before it in time and determinedly related to it. In other words, the will is not a spontaneous and independent thing leaning only against itself.

Santayana says, "Mankind and all its works are undeniably subject to gravity and to the law of projectiles; yet what is true of these phenomena in bulk seems to a superficial observation not to be true of them in detail, and a person may imagine that he subverts all the laws of physics whenever he wags his tongue, only in inorganic matter is the ruling of mechanism open to human inspection; here changes may be seen to be proportionate to the elements and situation in which they occur.... Physics cannot account for that minute motion and pullulation of the earth's crust of which human affairs are a portion. Human affairs have to be surveyed under the categories lying closer to those employed in memory and legend.... That this gulf is apparent only, being due to inadequacy and confusion in human perception rather than to incoherence in things, is a speculative conviction altogether trustworthy.... Now the human senses are not at all fitted to represent an organism on the scale of the human body. They catch its idle gestures but not the inner processes which control its action. The senses are immeasurably too gross. What to them is a *minimum visibile*, a just perceptible atom, is in the body's structure, very likely, a system of worlds, the inner catclysms of which count in

producing that so-called atom's behaviour and endowing it with affinities apparently miraculous. What must the seed of animals contain, for instance, to be the ground, as it notoriously is, for every physical and moral property of the offspring?.... Any one who can at all catch the drift of experience—moral no less than spiritual—must feel that mechanism rules the whole world."

According to Spinoza, that masterful combination of reason and intuitional insight, "A thing is said to be free (*libera*) which exists by the mere necessity of its own nature, and is determined in its actions by itself alone." If, then, men can attribute no reason for the willing of anything beyond the immediate cause, then the will is infinite beyond that cause; then the will is equal in power to God, in that He would have no control thereover and all the burden and responsibility of a choice, which may affect the lives of many men, is placed upon this will, infinite in its nature yet limited in its knowledge. It is not just nor right that God should place such responsibility in the ungoverned hands of ignorance. As God is just and righteous it follows "from these premises then, that men think themselves free inasmuch as they are conscious of their volitions and desires, and, as they are ignorant of the causes by which they are led to wish and desire, they do not even dream of their existence." It is then concluded (Prop. 48, Part II) "There is in no mind absolute or free will, but the mind is determined for this or that by a cause which is determined in its turn by another cause, and this one again by another, and so on to infinity. *Proof.*—The mind is a fixed and determined mode of thinking and therefore cannot be the free cause of its actions. It cannot have the absolute faculty of willing or unwilling, but in willing this or that, it must be determined from an infinite line of causation."

Dr. James says in one of his essays, "The sting of the

word 'chance' seems to lie in the assumption that it means something positive, and that if anything happens by chance it must needs be something of an intrinsically irrational and preposterous sort." But I confess I can not see that unless *chance* is *governed* (or as he says, "needs be,") by reason or law (a contradiction in itself) how the result of the comparatively few higher animal actions of the future could be anything but "irrational or preposterous." It is a case of to be or not to be. If "chance" is to be governed by reason and by law we may expect the world to continue a part of a universe in the future and if it is not, that it will become participant in a nulliverse. Regret for our past actions and therefore the wish that something might be otherwise takes place in every passing hour and is but a confession that had we been wise enough our act would not have occasioned regret as it would have been governed by that wisdom.

The distinct purposive intervention of the self-conscious ego is what should be designated as will, though the purpose and intervention be caused; it is purely voluntary to us and gives no feeling of oppression although in the true sense not "will." Therefore to say that you cannot perform will as I have re-defined it is untrue, for the memories and instincts—caused causes of the will, are a part and contained in yourself, i. e., to you the act is will. From a point of view outside of the self the ego is not responsible, but you are to yourself since the will is responsible for its conduct to the memories and instincts—the basis of the personality. Yes, and the responsibility is exactly fulfilled.

I can do no better than conclude my argument from the mental point of view with an illustration from the thoughtful pen of Thomas Huxley: "Suppose that an adult man, in the full rigor of his faculties, could be suddenly placed in the world, as Adam is said to have been, and then left to do as he best might. How long would he be

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left uneducated? Not five minutes. Nature would begin to teach him, through the eye, the ear, the touch, the properties of objects. Pain and pleasure would be at his elbow telling him to do this and avoid that; and by slow degrees the man would receive an education which, if narrow, would be thorough, real, and adequate to his circumstances, though there would be no extras and very few accomplishments. And if to this solitary man entered a second Adam, or, better still, an Eve, a new and greater world, that of social and moral phenomena, would be revealed. Joys and woes, compared with which all others might seem but faint shadows, would spring from the new relations. Happiness and sorrow would take the place of the coarser monitors, pleasure and pain; but conduct would still be shaped by the observation of the natural consequences of actions; or, in other words, by the laws of the nature of man. Nor should I speak of this process of education as past, for any one, be he old as he may. For every man the world is as fresh as it was the first day, and as full of untold novelties for him who has eyes to see them. And nature is still continuing her patient education of us in that great university, the universe, of which we are all members, nature having no Test-Acts. Those who take honors in nature's university, who learn the laws which govern men and things and obey them, are the really great and successful men in this world. The great mass of mankind are the 'Poll,' who pick up just enough to get through without much discredit."

I have quoted this at length because it so admirably conveys the meaning which I have tried to express in other words, and because it contains the foundation of my argument from the moral side of the question which is about to follow. In this domain the exponents of free will have considered themselves least needful of defence, but, as yet, I have not come upon any elucidation of this side

of the question which was exactly satisfactory to my demands. Either we are wrong when we blame, or God is immoral, and I greatly suspect that the fault is to be found in our lack of true moral comprehension, rather than in God. It then becomes my duty to whitewash the Devil, although to compel him to keep indoors is the work of the centuries.

Going to the foundations of morality must necessarily give us a truer conception of the import of things, and must also lead, by way of our determinism, to a rational, optimistic, trusting conception of the universe if that doctrine is to be entertained by us for one moment. Before the advent of man, it is easily seen that nothing was moral or immoral, for those terms are merely relative to our mode of thinking and arose with it. Nothing therefore is within itself bad or good and the words signify only the fulfillment of the demands of our nature upon phenomena or the lack of it. That which does not acquiesce to our demands is called bad or evil and if it is shown that our demands are the result of comprehensive reasoning, i. e., that we can see to what end the action or being is directed, and that it is evil (detrimental to happiness and well-being of men) then we have a right to a pessimism toward that universe which would produce good and evil in motley alternation. It therefore devolves upon us here to prove that the tendency of all phenomena is that which would secure the approbation of our moral nature if we could realize their end. Of course we can conceive what would be to us a perfect universe where all pain and evil had a good in its place, but I think we can not censure the scheme or entertain a pessimism if we find that all that is not good is productive of good, regardless of the conscious experience of the individual through whose suffering good is to be realized. Darwin truly states that no species or individual is perfect for its reaction upon its environment (whether of the com-

plex nature or of the continual change of the latter) and Höffding perceives in his "critical" realism the never ending progress of life toward that perfection. Man's becoming a social animal has raised the complexity of his environment a thousandfold, both in relation to the acts and thoughts of other men, and in the increased menace of disease. Now although not so potent a factor as it was thought at first, the law of the natural selection of variant forms, cruel in itself, *has been* the sole means toward that good end, our mind. She selected those who gained and retained from dear experience (a seeming evil) the requisite knowledge. Although the weaklings and the deficients may have been fostered by an unexacting environment and the more fit cut off by accident, yet, in the first case, if the environment remained and nothing stepped in to improve the unfortunates and they generated degenerates which were still fostered by easy surroundings, the time came when the environment changed and their extermination proceeded. If the fit were plucked by accident, and it was exceptional, yet the weeding still went on until ones of just as high a level of fitness were produced. Nature works slowly now through painful education (each age building upon the knowledge gained by the preceding ages from their diligence and to a less extent their lack of it and mistakes) and reaching a higher social scale, after having gone as far as possible with painful extermination, (natural selection has caused reason which has supplanted instinct and selection as factors in our development) toward that consummation, the craving for which has produced the greatest hope of the human breast—knowledge and happiness (perfect adaptation to environment). Thus science confirms with positive knowledge that these beliefs which originated in the heart of primitive man, are not empty and groundless, but even confirms them along with that other—of an omnipresent, omnipotent God, which it

explains as a realization of the presence of The Law (of cause-effect).

The morphine fiend could not *help* himself because he was fated from eternity to pain; he had better not been born; but the law of life and of death cares not if a spark of consciousness suffer nor whether the spark know the consequences of its action or not. "Ignorance of the law is no excuse, and the wages of sin is death." Whether the victim is able to *help* his action or not the evil to him exists and although the act is an "evil" which, according to free-will, might not have been, it happened and the universe, be it a monism or dualism, possess we one element (causality) or two elements (causality and free-will), is responsible for its existence and the victim a right to pessimism as long as he regards himself. In either doctrine the only way out of the difficulty that I can see is to take the more comprehensive view, whether you be the victim or not. The invisibility and slow working of the evolutionary law (physical, mental, or social) may make this seem to be closet philosophy, but it is only when we make a retrospection of the ages that the great underlying influences come into broad daylight.

Let us take, by way of illustration, an event told by Dr. James. He says: "At Brockton, the other day, a man, to get rid of the wife whose existence bored him, inveigled her into a desert spot, shot her four times and then as she lay on the ground and said to him, 'You didn't do it on purpose, did you dear?' replied, 'No, I didn't do it on purpose,' as he raised a rock and smashed her skull." The Doctor remarks, "We feel that although a perfect mechanical fit to the rest of the universe, it is a bad moral fit and that something else would really have been better in its place." I do not say that something else would not have been better in its place, but his universe, as well as mine, must account for it and palliate the crime to us with a

reason which gains our moral approbation for its existence. I say that our moral view is not the true view, else it would allow the existence of these "evils." No evil is necessary, but as long as we are ignorant or governed by blind passion, we are not perfect in our environment, and the "evils" are bound to exist. The causes of such actions as these are unhealthy bodies, or minds which have not learned from their own or others experience (i. e., educated to a wrong environment), or who do not recognize the stronger demands of society or are guided by passion in lieu of the only legitimate monitor reason. Now, seeing these causes, could we blame the action of this Brockton man? Or could we blame the universe as immoral when it is necessary to evolve slowly into the social state and therefore actions such as this, reversions, come to pass in a state of society where they are immoral—individual strife was not immoral where individualism and natural selection were working as it led to a great good,—the physical and brain development of the race. But reason, experience, and social or moral education are taking place and the future we believe to promise a better condition. There is no more immorality in this mental reversion than in a physical reversion such as the famous Miss Julia Pastrana or a tailed boy. In fact, unless the experience of the past and our possession of reason counted for something in our life, I do not see how any social evolution, an optimistic view of the future, or any reason for our progress thus far can be had, since natural selection has become *nil* to us. The conscious experience of healthy men affirms the potency of reason and experience and as this is the surest of reality to us, I believe no doubt can be had. Even if there were no other palliation to our just desire for a rational and moral universe, the fact of the educational value of this Brockton example as an admonition to posterity would be sufficient.

But this action which I have just explained is rudimentary—the remains of a lower stage of mental evolution. The self-instinct was necessarily the first produced by natural selection and still remains with us although not playing such an important part. The preservation of the young or the family next arose and all actions were sacrificed to it; outside of the family the self-instinct was then guide. So lived our remote ancestors. But the development of the brain meant the birth of memory, comparison, and reason, for those individuals who possessed a little better memory of the consequences of actions were able to determine what would be the most probable result of one not yet performed and so could better serve their self or family instincts. Thus with the birth of reason, instinct became but a secondary factor, and our primitive ancestors, reasoning that a greater surety of food and protection was given by that social institution, the tribe, formed in those more efficient bodies, which had a greater scope of action than was possible for an individual. Natural selection still kept up a certain low standard within the tribe (by rivalry for females and by disease) and also outside of the tribe by selecting those tribes of the greatest population or best organization, thus spreading tribal formation over the continents. But to-day with the decrease of rivalry inside and outside of our social institutions, i. e., decrease of war, disease and personal conflict, natural selection has become almost inert. Our evolution—the evolution of our organization—is proceeding by means of the reasoning powers of man and by the necessity for social action forced upon him by his fellows. In early life he imitates and then sees the reason and expediency of social action. The self-instinct, the love instinct, the family instinct are here to stay, but as social evolution advances all actions are not caused by the first or as later by the first and second or as later when the field of action was divided among the first,

second and third, but the field of each of these instincts approaches its limits as the broader fields of service to the nation, and later to society, develop.

Thus we see that natural selection produced a high type of individual, produced the self-instinct, the family-instinct, and had a small part in producing the tribe semi-instinct. Then as reason also developed and partly by it, by instinct, or by imitation, men banded into nations, natural selection slowly subsided and organization and education appeared. The self-instinct of the leaders was limited by the strength of the demands of the others even as it is to-day, the difference being in the strength of the demands. People seldom obtain any more than they demand as self-instinct has the field (*produces* actions) until it is encroached upon by the stronger demands of our fellows. Thus the only moral law, and the only expedient mode of action for ourselves is to comply with the stronger social demands as far as they extend—not so far that we are overcome by the self-action of others. Thus we must fight individually to the extent that individualism is practiced by others and must conform to the growing demand for social action—but not as far as the new twigs which must find nourishment and grow before they will bear our weight. The cause of great suffering has been and will be, (until the limit—utilitarianism—is reached through education) in social evolution, in determining how far, in regard to one's self, social action encroaches our field of expedient self-action. In the most successful lives this dividing line is more approximately determined, and those are unfortunates, who from lack of observation or foresight act either as the criminal, robber, small tyrant, etc., (too much individualism) or such few and unnatural men as Timon of Athens of whom it could be said:

> **"Poor honest lord, brought low by his own heart,**
> **Undone by goodness! Strange unusual blood,**
> **When man's worst sin is, he does too much good!"**

They are to be pitied, but that which caused their action cannot be censured as immoral because it is a necessary accompaniment of the individual-social metamorphosis, and all admit that the end of social evolution is one of the greatest goods attainable by man.

At the present day an excellent example of this is afforded by the action of Germany in European affairs, and is applicable individually as well as nationally. Germany has asserted her self-rights as far as possible. She has exacted Alsace-Lorraine from France and is now endeavoring to shut her out of Morocco. It is a case of get as much as you can without burning your fingers. Now were England, France, and Russia to form a coalition, a strong demand would be created and, being expedient, Germany would have to comply with it. This is of course explaining the extreme expedient selfish case as it exists to-day. But there are others against whom there is not so strong an individual competition and who then can comply also with the lesser demands of society. As social evolution progresses these necessarily become greater in numbers and the evolution gains increasing force as it advances. As the child's first social acts are imitative and educationally induced and as later he sees the expediency of social institutions and demands, so progresses his moral education. And if he has the self-instinct strongly developed, its field of action in him will be limited only by the strongest and most immediate demands of society—demands which require the minimum amount of social action only, and he will not contribute to social progress. But those in whom the instinct is not of such force or who have been educated in highly organized communities, do not stop social action and revert to self-action only at the strongest demands of society, but comply with the lesser demands; themselves create lesser demands and strengthen the pre-existing ones, so that the social evolution of any community or

people depends on the number of this type of individual that it contains—if the self-actors predominate evolution would necessarily tend to revert to the remote unsocial period and vice versa. The great factor in producing the less selfish actors in the majority is, that once headed in the direction (usually by education) like habit, the tendency is to let the field of self-instinct be encroached upon gradually more and more (of course retaining as much of the instinct as is required by expediency to combat with the amount of self-action of others at the stage of evolution of the time of the individual). Thus the field of social action widens and limits that of self-action. New demands are created, by a majority; the former weak ones strengthened, and the strong ones are become a matter of course and habit.

It is apparent from this how any set rule for moral action has only been valid for the state of society at its birth, and how in order to lead the most satisfactory life we must comprehend (approximately) the existing state of social evolution—must observe and follow the amount of social action that can be indulged in without neglecting the individual action necessary to maintain one's self. Thus utilitarianism in being the consummation of moral or social evolution—all actions for the good of society and the maximum individual welfare possible for all (the welfare of society's individuals being its own) is not a fit "working hypothesis" to-day as a certain amount of self-action must be mixed with the social. It is, I believe, the goal of social evolution—distant, undiscernible, on the other brow of the earth—and we know the earth is round. Utilitarianism, service substituted for gain, thus seems the far off end of moral action.

I can in no way agree with M. Elie Metchnikoff, who, after showing the insufficiencies of the moral doctrines of Kant and Spencer says, "The ideal will rather be that

of men who will be self-sufficient and who will no longer permit others to do them good" — in other words the super-man of Nietzsche. He, a biologist and scientist, fails to scan the field of organic development and does not see that organization is the keyword to all progress in that field. The organization of "chromideals" into cells; the organization of cells into communities or organisms and lastly the organization of organisms into what we call nations and states. The key-word to organization is not self-sufficiency but specialization, cooperation, reciprocal action. The cells perform different functions and loyally work with the welfare of all the other cells (the community) in view, and the organism or community can function where the single cell could not. The analogy is complete. He fails to see that in order for the family to exist, one member must procure food and protection, one must raise the offspring, and the offspring when independent can then become the head of another family (it being necessary for the higher action of the animal that its infantile development be longer). In the tribe some must procure food, others make implements, others protect, etc., in order that individually the tribe may better live and function in accordance with a more complex environment. Would this not be a low social state if each individual had to grow or hunt his own food, manufacture his clothes, his house, his vehicles, etc.? He would be self-sufficient and no one would be doing him good!

The relations of the part to the whole in any highly specialized society are analogous to those of the vital organs to the human body. There is paralysis throughout the system when its functions are interrupted. The lower forms of life are so simple that you cut and subdivide them at will without any impairment of vitality, but as organization develops, with a circulatory system and coordinate functions for the several parts, their independence is lost.

And so in a primitive society the individual is comparatively independent, but as organization takes place and specialization proceeds and the exchanges of civilized life develop, the well-being of the individual becomes more and more dependent upon his cooperation with the other individuals. "Our civilization is based upon the division of labor. Its industrial efficiency, its wealth of production, its comfort and luxuries and variety of opportunity, are the results of cooperative effort. If each member of the community, instead of supplying his own wants, devotes himself to one thing and all exchange the surplus products with each other, the sum-total of their production and possessions is increased." Specialization and not self-sufficiency is the first word in organization, civilization, and social evolution.

Society is automatically regulated, for each man will select as his vocation that mode of action for which society pays most and which he believes himself capable of fulfilling, i. e., to him the strongest demanded (highest paid) mode of action. And according to his ability will he succeed in supplying the demand or descending to a position where he can. To trace the demands of society upon the individual, is to trace the social and moral evolution of the race.

I can see nothing but benefit and increase of happiness from the struggle of the old with the increasing new idea of social duty and in the unhappiness, pain, and sorrow caused by the non-conformity of those unlucky individuals who lacked the wisdom to obey the demands of society as far as these went, or who disregarded the necessary, individual self-action for their happiness in that state of social evolution. The battle has brought and is bringing our more complete organization and individual specialization, and hence greater individual safety from disease, from improper education and from all such mistakes and imper-

fections as now exist in our governmental and labor organization. The mistakes are a benefit to posterity as it learns from them what should be built upon the present inherited foundation to further the completion of the structure.

So I have shown the reasons, the why, the by-cause of our social actions—which form a great percent of all our actions, and have also shown therefore that they are no less automatic than the others. And not only that but I have palliated to our demands for a completely good, unsullied universe, the number of so-called evils,—the sorrows and pains, which have arisen along with the social evolution as well as those which have arisen from the physical evolution.

It is asked, what is the meaning, the import, the purpose of it all, why the necessity of this development? I can only answer, the universe is infinite. What could be the purpose of the purpose, or the import of the import? Were matter absolutely dense—without motion, we would have no problem; but change is the second most apparent phenomenon. There can be but one kind of change and that is of the position of matter. This may be resolved into molecular and molar motion. If a change in the kind of motion is made it is in the cycle of molecular to molar and by contact of bodies back into molecular motion. There is no purpose, that is too human a mode of thinking. There is but one possible process and that is change. In the universe existence and necessity are the factors; they are not finite as the mind, but free—on account of themselves alone. I sit and watch the development of a crystal—of which we are the molecules, our cells atoms, and our "chromidials" ions. The change of this crystal is molecular into the more substantial molar state accompanying and a part of the earth's change from nebulous to a more solid condition. There is as much import in our develop-

ment as the development of a grain of salt from solution, and the performance of the experiment is a show continuous. So much for the metaphysics of the question.

I have now, I believe, covered the entire scope of phenomena, have shown the reasons,—the causes of all actions, individual and social, and have shown how each leads life on to "a consummation devoutly to be wished." Thus the dread figure of "evil" has been exposed as a negative quantity while we admit and try to exterminate the to us evil. I have shown that we can blame nothing and that an optimism concerning the universe and its automatism is entertainable. Viscount Amberly has written, "Not in so slovenly a manner has the work of nature been performed. We are no more free to disturb the harmony and beauty of the universe than are the stars in their courses or the planets in their orbits . Our courses and orbits are no less fixed than theirs, and it is but the imperfection of our knowledge, if they have not been and cannot yet be discovered. But it would be a lamentable blot upon a universe, where all things are fixed by a law 'in whom there is no variableness nor shadow of turning' were there permitted to exist a race of creatures who were a law unto themselves." It is already recognized that knowledge repays a hundredfold the sweat that it cost us in this martyrdom of man, for we are thereby enabled to govern our future actions with greater wisdom and with more perfect reasoning, so I need not lay so great stress upon the almost omnipotence of the environment, the education of us all.

Thus, in the belief that "*Alles verstehen ist Alles dulden*," I widen my moral horizon from that of Dr. James, and find no phenomenon caused by that law-perfect-in-itself: cause-effect, which is not perfect mechanically and morally. I make suffering a good and destroy the word *evil*. Concerning the necessity for "evil"; there is no necessity and its existence is only caused by our imperfec-

tion, our ignorance. I no more regret the above incidents than I commend one of the opposite character (except for purpose of encouragement) or blame hydrochloric acid for acting upon zinc. If they say, well then there is no use in our trying, things *will* happen as set from eternity, I say, unless you do act according to that necessary instinct and competent memory you will justly become a victim and you or your life, if nothing of an opposite influence affect you or it, will justly become martyrs and perish in the cause of good. Nature cares nothing for individuals and it is the individual's self-instinct which has brought the free-will and immortality doctrines into being. "The optimism of scientific minds rests in the belief that upon the physical plane—the development of bodily vigor, or upon the intellectual plane—making him capable of reasoning and thinking for himself, or upon the ethical plane—making him a useful, trustworthy human being, all dependent upon beneficial heredity and educational environment, that mankind must be strong, able and free, and that we shall not dwindle into physical weaklings, intellectual nonentities, or spiritual slaves or fanatics." Munro continues, "Life consists in the free exercise of our faculties and happiness in the successful performance of duty and achievement." Indeed I am sure we *can* rely upon that factor which exterminates human inertness, and without which I can see no advancement, no cause for the struggle and no justification of evil to our moral natures.

Some say that the effect of this belief on them would be a feeling of a weight and pressure of the rule of mechanism, that they must feel free in order to remain happy and that there is something uncanny in regarding living creatures as mere complicated machines. These are certain preconceived ideas, arising, not from a change of belief induced by reason or by considering, as I have shown, that the will to us exists, but from a certain fear of the un-

accustomed caused by the absence or removal of a belief which had become a habit. Many peoples have lived happy with no feeling of oppression and been fatalists,—such as the old Anglo-Saxons and their *wierd* or fate, the Arabians and Persians who saw in all that took place the inevitable will of Allah, or, in more recent times, the Calvinists and others who betook themselves to this belief as the great and only consolation against the wrongs and injustices of the world. They were taught the belief; it was a part of them the same as the idea of free willing is a part of the majority of people to-day and so the opposite doctrine repulsive. We are thus human. It is a simple matter of attaining the correct attitude of mind and accustoming oneself to the idea, which is facilitated by the fact that will to us exists and that mechanism is more rational, more truthful, and more easily conceived.

A few more remarks will conclude all that I have to say. The belief that events are determinedly related to the condition of things immediately preceding them, is now held by all important thinkers in respect to all kinds of phenomena except higher animal volitions. In each successive department of fact, conflicting modes of thought have receded and faded until at last they have vanished everywhere except from this "mysterious citadel of the will." Then if we have any regard for consistency, and any regard for what facts, so far as we can see, tend to state, it is without the least disturbance of our scientific conscience that we can hold, until otherwise proven, that man is only a more complicated and variously endowed automaton, physical causes solely determining his bodily actions; the molecular activities of his cerebrum producing the succession of his mental states; and brain changes the real origin of those movements he is accustomed to regard as expressing his feelings, or as executing his intentions, those feelings and intentions being the mere "concomitant symbols

in consciousness." That the universe ought to be rational is what these conscious feelings tell us, and I think I have ascertained that most rational conception, monism. Reason should be satisfied and I have shown that all things are governed according to that reason which actuates them. Knowing that we cannot *help* doing what our heredity and environment necessitates, I have inferred the direction that may be given to the whole course of a life by a little effort on the part of another to fit the man better to his surroundings and to insure his well-being. And lastly, the most important, I have shown that we may entertain an optimism concerning the universe, a view at once so necessary to our peace of mind and to our obtaining the best out of an existence where life must be thought worth the living and the struggle to repay its cost. In fact I see no reason why we should not welcome with open arms a conception so beneficial to the body, to the understanding and to the craving of the heart.

STEWART P. FOLTZ.

ASHEVILLE, N. C.

GELLERT'S PHILOSOPHICAL POETRY.

ADOPTED BY BEETHOVEN AS THE CONFESSION OF HIS RELIGIOUS FAITH.

BEETHOVEN was born a Roman Catholic and in his early childhood he received impressions exclusively of Catholic traditions, Catholic worship, and Catholic art. It must always have appeared to the boy that the Catholic church was the only religious institution. When he left the city of his childhood and youth whose government was in the hands of a prince-archbishop, one of the electors of the Holy Roman empire, he came to Vienna which is now and was especially in his days a typically Roman Catholic city. It is remarkable that under these circumstances he was not more limited in his religious conviction and art by the ecclesiastical influence which had a strong hold, for instance, on Liszt. Beethoven's religion had broadened under the influence of his acquaintance with other world-conceptions, and it appears that Gellert contributed most to the formation of his views.

Beethoven was a great reader, and we can trace the growth of his conceptions not only by the books he read but also by the very sentences which impressed him, for he had a habit of underlining what struck him forcibly, and thus we can trace his philosophical and religious development. Though he never broke away from the church, he broadened, and his general attitude was not greatly different from that of any other great man of his age. He

admired Goethe though the two men were too different in character and disposition to become friends.

Beethoven's religion was strongly tinted by the rationalism of the Kantian school. His God was not the miracle worker, not the God who had revealed himself exclusively to Jews and Christians, and yet Beethoven did not hesitate to lend his art to the composition of a great mass. He was too broad to reject the artistic conception of a religion the dogmas of which he had outgrown.

As a rule when people broaden they become narrow in the very field of their mental growth. They love to parade their breadth of mind by objecting to those forms which characterize the narrower views. Not so Beethoven. He did not frequent the church or attend service, but he did not hesitate, when the opportunity offered, to write a mass for his friend the archduke Rudolf at his installation as archbishop of Olmütz, utilizing the traditional form of service that was customary in the Roman Catholic church. But his composition outgrew the limits of its earlier form. It became a cosmic epic, a doxology of the Creator, a triumphal song of God's glory and a proclamation of his divine dispensation.

The composition of this *Missa Solemnis* is no longer ecclesiastical in style. It has become poetry, and as such the Roman Catholic mode of worship serves as the basis for the presentation of a broader theme. It is like a philosophical drama in music; it is the denouement of the entire world process, an anthem to the infinitude of existence and the victorious advance of evolution, a hymn to the world-order.

In this same sense we have to interpret also Beethoven's compositions of the six religious songs of Gellert. They are Protestant in tone and Protestant in the austerity of their devotion. Beethoven accepts them not in the letter of the word but more as an artistic attitude

to express his own sentiments. We cannot doubt that upon the whole he made the thoughts his own, and here in Gellert's songs, if anywhere, is expressed his own religious conviction. From the sentiment of the sixth of these songs, called "Penitential Hymn," the present generation has become estranged, and it will be difficult for us to understand Beethoven's attitude; but it will explain itself if we consider that Beethoven in his constant fear of appearing insincere frequently gave offense to his best friends, and then showed his regret by profuse acknowledgement of his mistake. These outbursts of temper and an ostensible show of discourtesy toward his very best friends, most of whom belonged to the highest circles of the Austrian aristocracy, are mainly due to his democratic pride and to the fear lest he depart from his ideal of independence. It was for the sake of the God within him that he was carried away to brusqueness and rude behavior, and he felt the adjustment had to be made with himself before God alone.

We here insert a translation of the six hymns of Gellert, following mainly the translation of H. Stevens. They read as follows:

PRAYER.

O Lord, thy goodness reaches far,
As far the clouds are guided;
By mercy crown'd, thy creatures are
With needful help provided.
Lord! my defense, my tower and shield,
To me a gracious audience yield,
Approve my supplication.

LOVE THY NEIGHBOR.

If one shall say, "I love the Lord,"
While yet his brother hating,

With mockers he shall reap reward,
God's truth abominating;
For God is love, and wishes me
With all on loving terms to be.

DEATH.

Life is ebbing fast away,
Hourly towards the grave I hasten;
Death may come without delay,
Let this thought my spirit chasten.
Man bethink thee Death is rife,
One thing needful is in life.

NATURE PRAISES GOD.

The Heavens declare the Lord's infinite glory,
The sea and earth sound forth his name,
And tell their origin's wonderful story,
Mark well, O Man, what they proclaim.
Who gave the numberless stars their existence,
Who calls the Sun from his abode,
He cames in brightness and smiles from the distance,
And like a hero keeps his road.

POWER OF GOD.

God is my song!
In strength he reigns victorious,
High is his name,
And all his works are glorious;
Earth, Sea and Heaven to him belong.

PENITENTIAL HYMN.

I.

'Gainst thee alone, God, have I sin committed,
And evil done in thy dread sight,
Thou seest my guilt for which thy wrath is fitted,
See, Lord, my woe and sore affright.

My piteous wail, my sighs are all before thee,
My tears of deep and bitter grief.
O God, my God, shall I in vain implore Thee?
How long wilt thou deny relief?

Lord, do not after my deserts reward me.
Chastise me not! Show me thy face;
I crave for thee! thy pardon, Lord, accord me,
O God of patience and of grace.

II.

O grant me early, God, thy consolation,
Oh Father of mercy, God of love,
For thine own name's sake grant my supplication,
Thou lov'st to bless from Heav'n above.

Let me thy pathway tread; let me be steady
In my obedience to thy word.
To do thy will I shall be always ready,
I am thy servant, thou my Lord.

Lord, hasten thou to shelter and defend me;
Thy light shall lead, point out the goal.
Thy helping hand, O Lord, thy helping hand extend me
And with thy comfort fill my soul.

PAUL CARUS.

CRITICISMS AND DISCUSSIONS.

BUDDHIST LOANS TO CHRISTIANITY.

WITH SPECIAL REFERENCE TO RICHARD GARBE.

In the October *Monist* Professor Garbe, of Tübingen, admits a Buddhist basis for the Christian legends of Saints Christopher and Eustace. In the early part of the same article he also admits Buddhist influence in the Christian Apocryphal Gospels, but denies it in the Canonical ones. I herewith submit two passages from the Gospel of Luke which appear to me to agree as closely with the earliest Buddhist texts as do the saint-legends admitted by Garbe.

The first parallel is taken from my now forgotten pamphlet of 1905, *Can the Pāli Piṭakas aid us in fixing the Text of the Gospels?* The second is from my *Buddhist and Christian Gospels,* as indicated in the first edition (1902) and partially printed in the third and fourth (Tokyo, 1905, and Philadelphia, 1908).

THE ANGELIC HERALDS AND THEIR HYMN.

Luke ii. 8-14.	Sutta Nipāto, Mahāvaggo, Nālaka-sutta (known only in Pāli, but with analogues in later Buddhist books).
And there were shepherds in the same country abiding in the field, and keeping watch by night over their flock. And an *angel* of the Lord stood by them, and the glory of the Lord shone round about them: and they were sore afraid. And the angel said unto them, Be not afraid; for behold, I bring you good tidings of *great joy* which shall be to all the people: for *there is born* to you this day *in the city of* David a Saviour, which is *Christ* the Lord. And this is the sign unto you; Ye shall find a babe wrapped in	*The heavenly hosts* rejoicing, delighted, And Sakko the leader and *angels* white-stoled Seizing their robes, and *praising* exceedingly, Did Asito the hermit see in noonday rest. [He asks the angels why they rejoice, and they answer:] The *Buddha-to-be,* the best and matchless Jewel,

swaddling clothes, and lying in a manger. And suddenly there was with the angel a multitude of the *heavenly host praising* God, and saying,

Glory to God in the highest,
And on earth peace, divine favor among men.

Is born for weal and welfare in the world of men,
In the town of the Sākyas, in the region of Lumbini:[1]
Therefore are we joyful and *exceeding glad.*

The parallel is further carried out in the narrative. The hermit, like the shepherds, goes to pay his reverence to the newborn Saviour.

Considering that between the Greek of Luke and the Pāli of the Sutta Nipāto there may lie some lost book, the words in italics are practically identical. The Pāli words *hita-sukhatāya* ("for blessing and happiness") are a convenient phrase, often recurring in the texts. We here translate them "weal and welfare" for the sake of poetic effect, but they mean much the same as the English phrase, "peace and prosperity." Now if Luke, or rather his Oriental intermediary, did actually use the Pāli poem, it is evident that omitting *jāto* ("born"), we find a very good equivalent of the line:

Manussaloke hitasukhatāya jāto,

in the line:

ἐπι της γης εἰρηνη ἐν ἀνθρωποις εὐδοκια.

It is thrown into the form of a Hebrew parallelism, in which peace on earth and divine favor among men are interchangeable terms. It is well known that the oldest manuscripts of the New Testament are at variance here over the word εὐδοκια. Some read εὐδοκιας (genitive) and then we must render: "among men of good will" (or the divine favor, i. e., the elect, as Alford says).

This is the reading of the Vulgate and of the English and American Revised Versions. It is because εὐδοκια in the Septuagint means so often the divine good pleasure that the Revised Version has "men in whom he is well pleased." But the old King James reading (following the *textus receptus* afterwards fixed by the Dutch printers Elzevir) is borne out by the analogy of all Hebrew parallelisms. This is therefore a passage wherein the Pāli Piṭakas can probably aid us in fixing the text of the New Testament.

This parallel is ignored by Garbe, though he mentions that of Asito and Simeon, which is connected with it in the Pāli. But the

[1] A pre-Christian inscription was lately discovered, marking the site of Lumbini.

Lalita Vistara and other late books relied on by Garbe, and by Sanskrit scholars generally, do not contain the Angelic Hymn. I admit the weakness of the Asito-Simeon parallel, when taken by itself; but its strength consists in its organic connection with the Angelic Hymn, both in Luke and the Sutta Nipāto.

In *Buddhist and Christian Gospels* (4th ed. only) I have shown that Luke's alteration of the Buddhist legends is no more than his alteration of the Synoptic tradition (Mark xvi. 7, compared with Luke xxiv. 6).

When all this has been studied as carefully as older points of Gospel criticism, the day will come when school-children will know that "Peace on earth, good will to men" is a Buddhist text.

THE LORD'S THREE TEMPTATIONS.

Luke iv. 1-3.	Classified Collection, Book of Temptations (Pāli and Chinese).

In the Wilderness.

And Jesus, full of the Holy Spirit, returned from the Jordan, and was led by the Spirit *in the wilderness* during forty days, being tempted of the devil. And he did eat nothing in those days; and when they were completed, he hungered.	At one season the Lord was staying in the land of the Kosalā, *among the Himālayas,* in a log-hut. While thus living *in hermitage retired,* the reflection arose within him: "It is really possible to exercise dominion by righteousness, without slaying, or causing slaughter; without oppression or the making thereof; without sorrow or the infliction thereof."

Temptations to Assume Empire and Transmute Matter.
(In different order in Luke and the Pāli.)

And the devil said unto him, If thou art the Son of God, *command this stone that it become bread.* And Jesus answered unto him, It is written, Man shall not live by bread alone. And he led him *up,*[2] and shewed him all the kingdoms of the world in a moment of time. *And the devil said unto him, To thee will I give all this authority,* and the glory of them: for it hath been delivered unto me; and to whomsoever I will I give it. If thou therefore wilt worship before	*Then Māro, the Evil One,* perceived in his heart the thought which had arisen in the heart of the Lord and he approached the Lord and *spake thus: "Lord, may the Lord exercise dominion;* may the Auspicious One exercise dominion by righteousness, without slaying or causing slaughter; without oppression or the making thereof; without sorrow or the infliction thereof." "What seest thou in me, O Evil One, that thou speakest thus to me?"

[2] Matthew has: *unto an exceeding high mountain* (thus agreeing with the Pāli idea of the Himālayas).

me, it shall all be thine. And Jesus answered and said unto him, It is written, Thou shalt worship the Lord thy God, and him only shalt thou serve.

"Lord, the Lord hath practised the four principles of psychical power, hath developed them, made them active and practical, pursued them, accumulated, and striven to the height thereof. *So, Lord, if the Lord desired, he could turn the Himālaya, the monarch of mountains, into very gold, and gold would the mountain be.*"

[Buddha replies:]

"The whole of a mountain of gold, of fine gold,
Twofold, were not enough for one;
Let him who knoweth this govern his life.
He who hath seen Pain and whence its rise,
How could such a one bow to lusts?
He who knoweth that the substratum of existence is what is called in the world 'Attachment,'
Let that man train himself in the subdual thereof."

Then Māro, The Evil One, said, "The Lord knows me; the Auspicious One knows me." And he vanished thence, unhappy and disconsolate.

Temptation to Commit Suicide.

(Continuous in Luke).

And he led him to Jerusalem, and set him on the pinnacle of the temple, and said unto him, If thou art the Son of God, cast thyself down from hence: for it is written,
He shall give his angels charge concerning thee, to guard thee:
and,
On their hands they shall bear thee up,
Lest haply thou dash thy foot against a stone.

Book of the Great Decease: Long Collection, Dialogue 16; Chinese, No. 2. (Three months before Buddha's death).

Now not long after St. Anando had gone, Māro, the Evil One, approached the Lord, and standing beside him, addressed him thus:

"O Master, let the Lord now die the death of an Arahat,[3] let the Auspicious One die the death of an Arahat: now, O Master, is the time for the Lord to die this death; and moreover this word was spoken by the Lord: 'O Evil One, I shall not die the death of an Arahat until my

[3] *Parinibbātu,* literally "become extinct," conveying the double idea of physical and passional death. See note in *Buddhist and Christian Gospels,* fourth ed., Vol. II, p. 99.

And Jesus answering said unto him, It is said, Thou shalt not tempt the Lord thy God.	monks and nuns, my laymen and lay-women become wise and trained disciples, reciters of the Doctrine, walking in the doctrine and the precepts, walking consistently, living out the precepts..... "And now, Master, [is this the case]. O Master, let the Lord now die the death of an Arahat, let the Auspicious One die the death of an Arahat; now, O Master, is the time for the Lord to die this death!" When he had thus spoken, the Lord said unto Māro, the Evil One: "O Evil One, be content; the Tathāgato's Arahat-death will not be long: at the end of three months is the time for the Lord to die the death of an Arahat."

The Devil Disappears.

And when the devil had completed every temptation, he departed from him for a season.	Classified Collection (in sequence above).

Here we have, in the Pali and the Chinese of the Classified and Long Collections, representing two Buddhist sects of great antiquity, the following root-ideas:

1. Appearance of the Tempter to the Saviour in a wilderness.
2. Temptation to assume empire.
3. To use mystical power to transmute matter.
4. To commit suicide.
5. Disappearance of the Tempter when foiled.

Now Luke has these same root-ideas, though expressed differently in the third case (or, in his text, the first): viz., the transmutation of stones into bread instead of into gold. Matthew also has them, but he interpolates Luke's third temptation (that of suicide) between them. I therefore give the text of Luke, because it agrees with the Buddhist association, as Luke so often does.[4]

It is imperatively necessary to study these parallels by means of their earliest sources; viz., the Pāli and Chinese Hināyāna texts

[4] See the article *Luke and Buddhism*, in the General Index to the fourth edition of *Buddhist and Christian Gospels*. Of course there is the possibility that the Temptation scenes of Luke and Matthew (they are not in Mark, though he mentions the Temptation) belong to a lost book whereto both are indebted. I believe scholars generally consider that these scenes were not in the Logia source. My own belief is that Luke was the first to introduce them, and the editor of Matthew adopted them from his text.

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on the one hand and the Greek Gospels on the other. Seydel made the great mistake of dealing with late books like the Lalita Vistara, without distinguishing its lesser value for the comparison. Even so learned a scholar as Garbe still holds to the Seydel tradition, and consequently makes short work of the Temptation parallel by quoting these later legends (*Monist,* October, 1911, pp. 517, 518).

I maintain that there is as much striking agreement between Luke and the Hinayāna texts as there is between the Jātakas and the legends of Saints Christopher and Eustace, except that the latter are much longer and furnish more details for comparison.

In the temptation story there is the same Christian coloring as in the saint-legends, and yet the root-ideas agree. The Christian coloring consists in making the Master quote scripture, whereas the Buddhist idea requires him to state some truth. Again and again in the Jātakas do we find the same magical efficacy ascribed to the calm enunciation of a truth which the Brahmins ascribe to the words of the Veda and the Jews to those of the Torah. In the Zend-Avesta the Tempter uses a similar sacred word, but, as hinted elsewhere (*Buddhist and Christian Gospels,* 4th ed., Vol. I, p. 106), the Mazdean temptation story is only like the Christian one in its theism and its quotation of scripture. The earliest account of the temptation of Zoroaster is in the Vendidād, and it consists of only one, viz., that of empire. Before the temptation the fiend makes a vain attack on the prophet's life, and after it the prophet declares that he will defeat the forces of evil by two things:

1. The eucharistic utensils and sacred drink;
2. A magical word taught him by the Godhead in a past eternity.

While all this is of fascinating interest to the student of religion and of the New Testament in particular, yet it is by no means so close to the Christian stories as are *the earliest* Buddhist ones.

The Classified Collection and the Decease Book represent home-grown primitive Buddhism. And with these does Luke agree rather than with the geographically and theologically nearer Zoroastrian account.

In two other cases does Garbe neglect important parallels from the Pali Nikāyas. On page 521 he gives us interesting evidence, from his Sanskrit reading, of the Hindu character of the idea of walking upon the water, and says (as since amended) that it "belongs not only to the India of Budhism, but to that of Brahminism also." He ought to have added that the power to walk on the water is among the gifts of a pious Buddhist, ascribed to him by Buddha

himself, in the sixth sūtra of the Middling Collection in the Pāli (No. 105 in the Chinese version of A. D. 397)—a Hindu book far older than the Brahmin Mahābharata (though not of course than its ancient nucleus).

Again on page 517 Professor Garbe says: "Christ fasts forty days *before* the Temptation, Buddha twenty-eight days *after* the Temptation." But in the thirty-sixth sūtra of the Middling Collection we read that Buddha fasted nearly to death before his illumination, and therefore before his Temptation, which latter occurred after he was Bhagavā (the Lord).[5]

No one who studies the *Periplus of the Erythræan Sea,* a captain's log book of the first century (now newly translated by Wilfred H. Schoff of Philadelphia) will be able to agree with Professor Garbe (p. 524) in his limitation of the probability of Indian influence on Palestine to later times. The Periplus agrees, for the sixties, with Strabo, who saw 120 ships ready to sail from a Red Sea port to India in the twenties of the first century. And, as Wilfred Schoff has shown in his article on another page of this issue, the Roman Empire had a sort of Indian craze at that very time.

In *Buddhist and Christian Gospels,* the Lalita Vistara and other later books are treated in the Appendix as "Uncanonical Parallels," while the body of the book deals with canonical parallels, translated from the Pāli texts by myself and compared with the Chinese version of another ancient recension of the Buddhist scriptures (the Hindu original of which is lost) by Professor Anesaki of Tokyo.

When Rhys Davids's *Buddhist Suttas* (Sacred Books of the East, Vol. XI) were sent me by my bookseller in 1881, I found therein a vigorous protest against any attempt to trace Buddhist loans in the New Testament. This made a great impression upon my youthful mind, and acted as a deterrent in that direction until nearly the end of the century. Then, in 1899, Rendel Harris astonished me by postulating a Buddhist influence in the Acts of Thomas and (save the mark!) in the Gospel of Luke! I was stunned at first, then rallied myself and returned to my old objections. During the next seven years, however, deeper research caused me to change; and when in 1906 I observed the double quotation in John,[6] I admitted that here at least was tangible influence. It was anent the essay which I then wrote that Rhys Davids said

[5] Samyutta Nikāyo, already quoted. Had the Temptation occurred before the Illumination we should have read *Bodhisatto.*

[6] See "Buddhist Texts in the Fourth Gospel," *Open Court,* May, 1911.

to me: "The evidences in favor of intercommunication are growing every day." (I asked his permission to quote this, and he granted it). Paul Carus, in *The Open Court,* October, 1911, has adduced a remarkable picture from a Greek vase, portraying a goddess with water for her lower body, and he thinks that both the Buddhist and Johannine texts may be dependent upon some such ancient idea. So they may, but the strength of my case lies in the fact that the Fourth Gospel's express quotations from sacred literature (*Law* and *Scripture*). Instead of admitting that the quotations are from the Buddhist writings, where I have found them, several of my critics prefer to ascribe them to some lost apocryphal Jewish book. But the time is rapidly passing when scholars will feel compelled to adopt any hypothesis rather than admit the greatness of ancient India and the supremacy of Buddhism which, at the time of Christ, was the most powerful religion on the planet and the dominant spiritual force upon the continent of Asia.

In *Buddhist and Christian Gospels* (4th ed., Vol. II, p. 237) we read:

"A collection of [uncanonical] parallels would probably suggest a Christian influence upon later Buddhism; and indeed we know that, in the eighth century, a Chinese emperor had to forbid the two religions to be mixed. (See Takakusu's note in his *I-Tsing,* Oxford, 1896, p. 224.) This whole field needs very careful working, more than I am able to give."

Two Anglican clergymen, the late Samuel Beal and Arthur Lloyd recently deceased, have maintained this position. The fact is that after Kanishka's Council a new type of Buddhism, predominantly Mahāyāna, gradually supplanted the earlier. This new type was largely foreign, as the primitive type had been native Hindu. Before the Scythian invasions at the end of the first century, the Buddhism of Asoka, with its Pāli texts, had been in the ascendant; and as, in the first century, Christianity was in a formative stage, while Buddhism was settled and aggressive, the loans went from east to west. But afterwards there was a change. In the first place, a different race of sailors appeared in the Red Sea ports,[7] bearing with them the newer Buddhism which they themselves were helping to modify; and, secondly, Christianity itself was becoming a rival to Buddhism, and was beginning to assert itself.

It may be that Buddhism influenced the Roman Empire by

[7] I owe this information to Wilfred H. Schoff, translator of the new edition of the *Periplus.*

means of intermediary books, such as that of Elkesai which had a confessedly Buddhist origin ("Seres of Parthia"); but I maintain that the Nikāyas of primitive Buddhism were strong enough to make themselves felt more directly. In A. D. 149 a Parthian prince headed a long series of scholars who translated them into Chinese; but Buddhism had been established in the Greek empire (*Yonaloko*) since the third century B. C., and was quoted, chapter and verse,[8] by a Greek king, Menander, in the second. Now, the Chinese began to translate Buddhist books immediately upon that religion's introduction into their country in the sixties of the first century; and after a generation or two of translating manuals, lives of Buddha etc., they spent three centuries (circa 150-450)[9] in translating the Nikāyas (or Agamas). Were the Greeks less curious than the Chinese? Had not they also begun to translate the books they admired long before the time of Christ? My thesis is this:[10]

While a religion is in its formative stage, its founders take ideas from their environment, and especially from any system of thought that is paramount, whether in their own country or in those wherewith they have intercourse. But, once knit together, and moving by its own momentum, a religion can no longer add to its primitive documents, though it may give way to new influences in later sectarian developments.

The thesis applied is this:

During the first century Christianity was in its formative stage, and was influenced by the Old Testament, the Greek mysteries, the Philonic philosophy and by Hinayāna Buddhism. After the first century Christianity was strong enough to influence another religion in its formative stage. And such was Mahāyāna Buddhism, which was, in fact, a new religion, with new doctrines and new sacred books. At the same time, Hinayāna Buddhism still existed, and indeed its votaries often cultivated the Mahāyāna too. Consequently there could be and there was a complex interchange between Christianity and Buddhism, both of them giving and taking. But the earliest interchange was when the Hellenizing Evangelists Luke and John borrowed some minor features from the Hinayāna Nikāyas, then in the ascendant.

Before closing, let me add a note on the Wandering Jew legend

[8] So in the Pāli, though Chinese versions do not bear it out.

[9] Anesaki in *Transactions of the Asiatic Society of Japan*, 1908, p. 15.

[10] See my remarks on the Imperfection of the Record (following Darwin) in *Buddhist Texts in John* (2d ed., 1911, p. 27).

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among the "Uncanonical Parallels" in my *Buddhist and Christian Gospels.* I lately learned that Sabine Baring-Gould in 1866 pointed out that the germ of the legend is actually found in the canonical Gospels:

Mark ix. 1: "Verily I say unto you, There be some here of them that stand by, who shall in no wise taste of death, till they see the Kingdom of God come with power."

Let me repeat what I said last May in *The Open Court,* and which Professor Garbe does me the honor to quote: *Each religion is independent in the main, but the younger one arose in such a hotbed of eclecticism that it probably borrowed a few legends and ideas from the older, which was quite accessible to it.* The loans are not an integral part of primitive Christian doctrine, as I said in my Tokyo preface (1905), but lie outside of the Synoptical narrative, and occur in the two later Gospels of Luke and John, both open to Gentile influences.

Even now I only put forth these parallels upon the same footing as Gaster, Speyer and Garbe's Christopher and Eustace; and if the scholars of Europe and Asia finally decide that they are wrong, I shall withdraw my venture with a good grace. But if this great admission of Buddhist influence upon the Christian Apocryphal Gospels and the Eustace and Christopher legends receives its "brevet of orthodoxy," the next step will lead a new generation of scholars back to the canonical Gospels and the canonical Nikāyas.

ALBERT J. EDMUNDS.

PHILADELPHIA, PA.

FIRST CENTURY INTERCOURSE BETWEEN INDIA AND ROME.

EDMUNDS VS. GARBE.

In *The Monist* for October, 1911, appears a paper by Prof. Richard Garbe of Tübingen entitled "Contributions of Buddhism to Christianity," the essence of which is that common material is found in the Apocryphal writings of both religions, but that no connection can be proved between the Canonical texts, and that this is due to the fact that active intercommunication between India and the Mediterranean did not exist until the second century, or, as Professor Garbe puts it, "Buddhist influence might have penetrated to Palestine by way of Alexandria, but still more probably by way of Antioch in Syria, but they" (that is, writers pointing out similari-

ties) "are not apt to raise this possibility to a serviceable degree of probability for as early a period as the first post-Christian century."

In thus denying the existence of a rapidly growing and very important stream of commerce between India and Rome, it seems evident that Professor Garbe has overlooked historical facts which, if duly recognized, may compel him to revise his opinion in this matter as he changed his mind in regard to the migration of the fish-symbol from India to Rome.

The incontestable facts of history are that a large Indian influence and an active commerce existed as far as the Mediterranean coast of Syria soon after the conquests of Alexander, and that the conquest of these territories by Roman armies ending in the public triumphs of Pompey the Great, created in the Roman capital a craze for Indian products and luxuries of all kinds which during the actual lifetime of Christ had become a serious problem to the Roman government, leading to numerous efforts at discouragement of the taste for Eastern luxuries which was draining the Empire of its resources. This craze met with a temporary check at the death of Nero. It regained full intensity under Trajan and Hadrian, and was again in a decline during a considerable part of the second Christian century, reviving during the reign of Commodus, and again more seriously declining with the failing powers of the Empire. The existence of this craze for Indian imports and of the substantial remittances of gold coin required to balance the trade, may be surely proved by the hoards of Roman coin unearthed in Southern India and catalogued by the Government Museum at Madras; in which these fluctuating eras of trade prosperity and depression clearly appear. Instead, therefore, of the creation of a new import trade from India in the second century, as Professor Garbe asserts, the most active trade was in the first half of the first century, with two revivals at the beginning and the end respectively of the second; and the drain of specie from Rome to the East had set in even before the birth of Christ.

Space forbids a statement in detail of the almost innumerable facts existing to support the foregoing statement. The following may at least serve as suggestions.

Alexander married a Persian princess, but numbers of his officers took Bactrian and Indian wives.

Greek colonies were established by him along the entire Indian frontier, and colonies of his newly established Indian subjects were

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similarly established nearer Greece. A Greek dynasty ruled in Bactria after the Parthian revolt disrupted the Seleucid empire, and one of its rulers, Menander, powerfully influenced the spread of Buddhist thought through the Greek-speaking world.

A Greek ambassador at the Maurya court, Megasthenes, wrote a detailed account of its customs, its Brahmin religion, and its capital Pātaliputra; which was widely read and commented upon for centuries.

The conquest of Judea by the Persians and the destruction of the Persian empire by Alexander, reduced the force of Judaism and Mazdaism as world-religions, while the exodus of the Greeks into the East broke down what was left of the distinctive Greek religion. There existed then no faith strongly upheld in the Eastern Mediterranean basin from the third to the first centuries B. C.

Two generations after Alexander's conquests, the Emperor Asoka established Buddhism as the state religion of India, and in his second edict, preserved to us in a rock inscription, he mentions the sending of envoys to all countries with which he entertained relations; particularly mentioning "the dominions of the Greek king Antiochus, and those of the other kings subordinate to that Antiochus." This ruler is identified with Antiochus Theos (B. C. 261-246) in whose capital of Antioch these Indian envoys, physicians and missionaries, for they seem to have held that triple character, were received. In the capital of that ruler who profaned the Jewish Holy of Holies in order to set up the worship of himself, the Buddhist faith was preached by men sent from the head of the Buddhist organization, the ruler of the richest, most powerful and most populous empire in the world at that time.

During the better days of the Seleucidæ, overland communication between India and Syria was unhampered, and there is every indication that it carried an active commerce. The fall of the Seleucid power and the rise of the Parthian monarchy interposed a fiscal obstruction which the Greek rulers in Egypt, the Ptolemies, quickly turned to their advantage. By the establishment of ports on the Red Sea, Egyptian shipping was enabled to trade in the Gulf of Aden and obtain Indian merchandise with less transshipment than had formerly been made, and the opulence of this trade is vividly described by Agatharchides, writing in the closing years of the second century B. C.

For two centuries following Alexander's death we may assume that the Indian trade went no further than the Eastern Mediter-

ranean; but the rise of Rome as a world-power, dating finally from the sack of Carthage and Corinth in 146 B. C., brought the Romans into active trade with the Levantine ports, as evidenced by the growth of piracy in that region, preying on the Roman ships. Pompey's contributions to the Roman state were the suppression of the pirates and the conquest of the Levant; and in his triumphal processions, which are repeatedly mentioned by Pliny in his "Natural History," all the more precious varieties of Indian merchandise were exhibited and brought into popular demand. This point is of importance. Two generations before the birth of Christ the spoils of a conquered land resulted in a fashion for the imports of that land rather than for its own products: for the Indian goods transshipped at the Syrian ports, rather than for the products of Syria itself. The Indian trade had become Syria's richest asset.

The same facts are in evidence upon the conquest of Egypt and the incorporation of the Alexandrian trade into the Roman fiscal system. Primarily grain was the staple export from Egypt to Rome, but the more profitable trade consisted in the incense of Arabia and the gems and spices and textiles of India.

In 22 A. D., in a letter from the Emperor Tiberius to the Roman Senate set forth by Tacitus in his "Annals," the growing drain of specie is pointed out and a remedy demanded. "How," said the Emperor, "are we to deal with the peculiar articles of feminine vanity, and in particular with that rage for jewels and precious trinkets, which drains the Empire of its wealth and sends, in exchange for baubles, the money of the Commonwealth to foreign nations; even the enemies of Rome?"

The geographer Strabo, writing in almost the same year, records having seen a single fleet of 120 ships about to start by the favorable monsoon from an Egyptian Red Sea port to India. Two generations later, according to Pliny, the unfavorable trade-balance had grown more serious still; as he says "in no year does India drain us of less than 550,000,000 sesterces, giving back her own wares, which are sold among us at fully 100 times their first cost."

550,000,000 sesterces in those days was a very considerable sum. In modern valuation it would approach $25,000,000, and this was the state of affairs existing at the end of the reign of Nero. Can one imagine a modern trade requiring so enormous an export of specie without a corresponding influx of merchants, bearing ideas no less than goods, from the producing to the purchasing market? This condition is indeed set forth with sufficient exactness

by the writer of the Apocalypse, where he describes, under a veil of fiction, the burning of Rome and the ruin that thereby came upon "every ship-master and all the company in ships, and sailors, and as many as trade by sea," while of the merchandise they handled are specified numerous Indian products, precious stones, pearls, silk, ivory, fragrant wood, iron (Indian steel was known even to the Greeks), cinnamon, odors, ointments. This was in 64 A. D. A year or two before, according to Pliny, at the funeral of Nero's consort Poppæa, there was burned a store of Eastern spices representing a year's imports and valued at millions.

The unknown merchant of this same period who has left us that interesting log of his trading voyages from Roman Egypt to India which we know as the "Periplus of the Erythræan Sea,"[1] enters more specifically into the various articles dealt in and the marked growth in the trade. Briefly following him along his voyage, at the lower western shore of the Red Sea were imported Indian iron and steel, Indian cloth, muslin and lac. On the opposite shore, at the Arabian side of the straits, was a special port established for incoming Indian ships, which were apparently forbidden to trade by the Arabs' port of Muza. On the outer coast, which we know as Somaliland, Indian cinnamon was found and ships of larger size were now required to handle it. Other Indian gums are specified, among them gum *dammar,* and an Indian remedy for tropical disorders, *macir,* which does not again appear in western commercial annals until the days of the Portuguese. At Cape Guardafui was a regular trading rendezvous to which came numerous ships from the Gulf of Cambay bringing cereals, clarified butter, sesame oil, cotton goods, and honey from the reed called "sacchari"; the first known record of sugar as an article of commerce.

On the southern coast of Arabia were two ports at which Indian shipping regularly called. At the one Roman coral, tin, copper and storax were transshipped for the Indian trade, and at the other, more to the east, Indian shipping often wintered. Proceeding with our merchant to the mouth of the Indus, we find these same Roman products recorded among the imports of Northwestern India including, strange to say, Italian wines, preferred to the Syrian, or Arabian; all of which were imported. At the port of Barygaza in the Gulf of Cambay, the newly established Saka government

[1] A new translation, with learned notes, of this document is listed by Longmans for 1912. The translator is the writer of this article.—Ed.

maintained a regular system of pilotage which was necessary to avoid destruction of foreign vessels by the tremendous tides of that estuary. These pilot-boats coasted the shores of the Gulf for 100 miles outside the port, and our merchant records that both Greek and Arabian shipping was guided by them. Here he found among other things, spikenard, highly treasured in the ointments of the time as appears in the Gospel of Mark, chap. xiv. 3-5; and more important still, murrhine, that Indian carnelian, its colors heightened by slow heat and shaped into drinking vessels for which, according to Pliny, fabulous sums were paid in Rome. Petronius broke one of Nero's basins valued at 300,000 sesterces, while Nero himself paid one million sesterces for a single cup. Here at Barygaza were also imported for the Indian markets Italian wine, copper, tin and lead for the coinage of the country, coral and topaz, storax for the Chinese trade, glass, gold and silver coin on which there was a profit when exchanged for the money of the kingdom,—the Roman coinage being superior to the Hindu, which was of base metals only, while the Roman gold coin formed the standard of exchange for all the nations of India. Further down the coast in the back waters of Cochin and Travancore he found especially pepper and malabathrum (cinnamon leaves), on account of the great quantity and bulk of which our merchant tells us, large ships were sent to those ports, Greek and Arabian as well as Hindu. Here were found also great quantities of fine pearls, ivory and precious stones, beryls, diamonds and sapphires, and tortoise-shell, coming from as far distant as the Straits of Malacca in ships specially recorded as "of great size" in comparison with those Roman ships with which our author was familiar. In the adjoining nation, easily recognizable as the Chola Kingdom, whose capital Uraiyūr (Trichinopoly) is recognizable under the author's corruption of Argaru, were found in profusion all the merchandise sent from Egypt; while its ports were a center of shipping not only from Egypt but from the Ganges and Malacca. Here our author digresses to mention Chinese silk brought overland through Bactria to Western India for reshipment to the Roman empire, and among the exports from Rome to balance this trade is again mentioned "a great quantity of coin," fully supporting the testimony of the hoards unearthed in Southern India and recorded at Madras. The coins of Claudius and Nero are among the most numerous of all discovered.

The word which the author of the Periplus uses for the palm oil found by him at Zanzibar, was a word brought from India, the

Prākrit *nargil,* coconut. The most authentic information at the disposal of Lieutenant Speke in preparing for his expedition for the discovery of the sources of the Nile, was a map based on the Hindu Purānas, and setting forth information brought by these same Indian vessels found by the merchant of the Periplus on the African coast. These traders had penetrated the interior and knew of the Nyanza lakes, as the Egyptians did not. The facts already cited are surely sufficient to show a volume of trade not only internationally important, but so great and so one-sided as to be recognized as a serious menace to the prosperity of the newer, poorer, and less populous empire of the West.

Petronius, Nero's crony whom Pliny connects with the mad auction of murrhine cups, has left us *Trimalchio's Dinner,* that inimitable sketch of parvenu society in Rome at the middle of the first Christian century, in which it is mentioned as a matter of course that a rich man sent to India for so slight a thing as mushroom spawn. Pliny tells how Lollia Paulina, wife of the Emperor Caligula, wore at an ordinary betrothal entertainment emeralds and pearls to the value of 40,000,000 sesterces; "indeed, she was prepared to prove the fact by showing the receipts and acquittances." And he goes on to bemoan the prodigality in the use of Indian pearls by Roman women; "now, at the present day" (about 70 A. D.) "the poorer classes are even affecting them.... they put them on their feet, not only on the laces but all over the shoes; it is not enough to wear pearls but they must tread upon them."

The author of the Periplus tells how the Indian trade, as far as western shipping at least was concerned, used to be done in small vessels close to shore; and how Hippalus "by observing the location of the ports and the conditions of the sea, discovered how to lay his course straight across the ocean"—the monsoon being called the "wind of Hippalus"—so that from that time ships steered direct from the Gulf of Aden and Cape Guardafui to the ports of India, "holding their course straight out to sea with a favorable wind, quite away from the land." This discovery of Hippalus occurred in the time of Claudius, and the resulting increase of trade culminated under Nero. Pliny recounts the same story.

The distinction made by Professor Garbe between the parallelisms in the Canonical texts and those in the Apocrypha points to a period of change in the national and religious politics of India which is apparently not realized, and is yet of importance in the study of the interrelations between East and West. At the be-

ginning of the second century came the Council of Kanishka, the Scythian conqueror of the northwest, the second great Buddhist Council. The Scythians were looked upon askance by the native Hindus. It is recorded in the annals of the Andhra dynasty that after a victory over the Scythian or Kushan dominion, a memorial was set up at Kārli telling how the orthodox Andhra king had "destroyed the Sakas, Yavanas and Pahlavas, properly expended the taxes levied in accordance with the sacred law, and prevented the mixing of the four castes." A schism was thus set up in India, racial rather than religious at its root, which later expanded into the great division between the early Buddhist canon and its Mahāyāna corruptions. It was the earlier Buddhism which was carried to the Syrian coast by the messengers of Asoka. It was still a conservative Buddhism, but mingled with various central Asian religions, which was carried to the same region by the subjects of Kanishka; while the great changes of the succeeding centuries brought into Buddhism, no less than into Christianity, a mass of childish apocryphal legends which passed from one faith to the other in much the same way as the earlier ideas, which to some extent at least are found paralleled in the Canonical texts. The distinction is important; but it is a distinction based on changed national politics, rather than newly created trade, as Professor Garbe would infer. This change at the coming of the Scythian shipping into the Indian Ocean is vaguely indicated by Pausanias in a passage not usually understood, where he speaks of the Island of Seria (which was really Masira off the Southern coast of Oman) but which he confuses with the Seres of China. He tells us that "both the Seres and the inhabitants of the neighboring islands of Abasa and Sacæa [the modern Kuria Muria] are of the Ethiopian race. Some say, however, that they are not Ethiopians but a mixture of Scythians and Indians."

At that ancient meeting-point between the Nile trade and that of the Indian Ocean, the Abyssinian highlands, the author of the Periplus gives us the first mention of the Kingdom of Abyssinia, then newly established, and of its capital, "the city of the people called Axumites." The great series of monoliths at Axum dates probably from the first century rather than the second and shows orthodox early Buddhist influence rather than the Buddhism of later ages. James Fergusson's description of the great monolith has not been bettered, "the idea Egyptian but the details Indian, an Indian nine-storied pagoda translated in Egyptian in the first century of the

Christian era." He notes its likeness to such temples as the Bodh Gayā, and says it "represents that curious marriage of Indian with Egyptian art which we should expect to find in the spot where the two peoples came in contact and enlisted architecture to symbolize their commercial union." And so obviously Hindu a ceremony as the Brahman's investiture with the sacred cord is still preserved as the sign of baptism in Abyssinian Christianity.

Now the very existence of the Abyssinian state in the beginning was dependent upon the alliance of the Romans in Egypt, who encouraged its growth in order to counteract the Arabian domination of the Red Sea trade; and this was originally a matter of first-century diplomacy, culminating with the decay of the ancient Sabæan capital Marib, and the conquest of the Nabatæan kingdom under Trajan.

While these relations between India and the West were being developed, a similar connection was formed with the East. The silk-market of the world was in a fertile valley of the Pamirs, whither Chinese merchants brought their goods by the great *Pei-lu* or "Southern way" along the desert of Turkestan. Nomadic marauders hampered the trade, so that the author of the Periplus remarked of China that "few men come from there and seldom"; but the armies of Pan-Chao forged the last link of the great chain, and before the end of the first century communication was unbroken from the English Channel to the Yellow Sea, and the tin of Cornwall exchanged for the silk of Ts'in.

We are therefore forced to the conclusion that the middle of the first century of the Christian era was a time of unexampled commercial activity between East and West, that political turmoil both in Rome and India then caused a lull in this traffic, which did not fully revive until the later years of the second century, and that Professor Garbe's argument, in so far as it affects the general interrelation between Buddhism and Christianity, is to that extent in need of revision.

A Freeman could write "our business is with Europe, and with other parts of the world only so far as they concern Europe." And the Christian Gospels have been read with Western eyes. The Holy Land out of which they came has been conceived as a sort of Ultima Thule, beyond which lay a great void; the country beyond Jordan being remembered as a wilderness, wherein One was tempted of the devil. A barrier is thus set up and maintained, artificial and

without foundation, the defence of which some would assert to be a condition of right belief.

For some reason this type of critic would deny that an influx of new commodities carried with it a renascence of ideas, and would draw the old line about Christianity, limiting its environments to the country this side Jordan; inevitably admitting the larger expression which it received from the Gentile peoples of the northern coast of the Mediterranean, but ignoring that which came from the Gentile peoples beyond the Euphrates and the "Erythræan Sea." It is difficult to understand what is gained by so obviously tearing Christianity half out by the roots. The new faith reached out toward the East no less than toward the North and the West, and was so formulated as to be understood by all,—to be part and parcel of the intellectual environment of all. It would therefore be almost a matter of course that Christianity, making its appeal in the centers of trade, at the terminus of the great commercial highways from the East, should express its message in terms likely to be understood by those acknowledging Buddhism, the faith of the countries at the eastern terminus of those highways, and of all the world's faiths at that time, unquestionably the most influential.

Of lasting value, therefore, are all works which help to break down and destroy the ancient but artificial barriers between East and West; and of such works a very notable one is by Mr. Albert J. Edmunds, *Buddhist and Christian Gospels Now First Compared from the Originals* (Philadelphia, 4th edition, 1908-09).

Mr. Edmunds's work goes back to the age in which the Gospels were formulated, and reconstructs the background of world-thought and politics of which they have been so generally deprived.

It is necessary to a clear understanding of the Christian religion that a painstaking study be made of its points of contact with the Buddhist, and of the many thoughts which are their common property. Such a study can detract from neither faith, but must rather serve both, by showing more fully the human ideas and aspirations out of which they arose; by showing them to be living realities in the upward path of mankind, rather than abstractions limited each to its own area. It remains for the individual to make his choice between the two, but he must no longer be hedged in by an artificial barrier, which for centuries has separated peoples closely related at the Christian era, and now by the march of events, once more brought into contact. It is no longer possible for the Teuton to hold aloof from the Tartar, the Anglo-Saxon from the Japanese;

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mutual interest requires a closer understanding, a readier sympathy, and a fuller acknowledgment of common aspirations. Present-day commerce has its influence in this direction, and history likewise; but sympathetic comparison of the religions of the two races is among the most important of all such influences.

This work by Mr. Edmunds is therefore especially timely, and the ripe learning which he brings to this great subject assures its permanence.

Previous comparisons, such as those of Hardy and Seydel, had depended on translations and secondary authorities and had necessarily confused primitive writings with commentary and patristics, sometimes of late date; while Mr. Edmunds works with the advantage of an intimate knowledge of both the Pali and Greek originals. He has limited himself to parallels occurring only in the primitive writings of either religion, and his presentment is most convincing. The facts of history would naturally lead the open-minded investigator to look for a certain parallelism growing out of this ancient culture-field, but hardly to expect so formidable a list as 102 parallels of word or thought in the Canonical writings and 13 more in the books relegated to the Apocrypha, but of early date, in both religions. Furthermore, as Mr. Edmunds has shown in another place (*Buddhist Texts in John,* see also *Open Court,* May, 1911) Buddhist writings are actually twice quoted as scripture in the Christian Gospel of John. The proof of intercommunication is abundant.

Mr. Edmunds's comparisons provide a rich field of information for the student of comparative religion, and his conclusion is conservative enough to satisfy scholars of every kind. "No borrowing is alleged on either side—Christian or Buddhist. In these parallels we offer no theory but present them as facts. They at least belong to a world of thought which the whole East had in common."

Were it necessary, many other facts in the history of Syria and Palestine might be cited in support of Mr. Edmunds's argument. The Persianizing tendencies in the later Jewish church, due to the captivity in the Empire of Cyrus, are well known, while recent works by such British investigators as General Sir Thomas Holdich in upper India and Afghanistan, marshal abundant evidence of the eastern extension of the Assyrian Empire and actually of the settlement of Jewish captives in considerable numbers at the very gates of India. Here then was a central administration dominant from the Nile to the Indus seven centuries earlier than the period when

Mr. Edmunds seeks to prove active intercommunication. Six centuries before the same period, one of the last of the Pharaohs opened a canal from the Nile to the Red Sea to bring his country into communication with the Eastern trade in defiance of her Mesopotamian oppressors. Six centuries after the Christian era Buddhist and Christian legends were so mingled in Western Asia, that the Koran absolutely confused the two; while a little later in Eastern Asia a Chinese emperor issued an edict forbidding the same confusion then prevalent in his dominions.

It should hardly be necessary to recall that Palestine was the West-land of the Mesopotamian civilization just as India was the East-land; and that it was at the western rim of that ancient culture-field, and not from the Greek or Roman environment, that the Christian Gospels arose, just as it was at the eastern rim that the Buddhist writings were formulated. Without in any way assuming identity of origin or purpose, it would be strange indeed if there were not identity of expression and parallelism of thought between these two great Canons; and Mr. Edmunds's proof of that identity is a distinct contribution to human knowledge.

WILFRED H. SCHOFF.

PHILADELPHIA, November, 1911.

MR. BERTRAND RUSSELL'S FIRST WORK ON THE PRINCIPLES OF MATHEMATICS.

In *The Monist* for January, 1910,[1] Dr. Carus has criticized an article of Mr. Bertrand Russell's on "Recent Work on the Principles of Mathematics," published in the *International Monthly* for 1901. A copy of the article lately came into my hands, corrected in Mr. Russell's handwriting back again to what he originally wrote.[2] The editor or type-setter occasionally changed Mr. Russell's words to words which he considered more dignified, perhaps. Thus, the *International Monthly* makes Mr. Russell say[3] that in pure mathematics we "take any hypothesis that seems assuring, and deduce its consequences." Mr. Russell had written "amusing," and the substitution of "assuring" rather took away from the force of Mr. Russell's contention that in mathematics we are not in the least con-

[1] Vol. XX, pp. 46-63.

[2] Mr. Russell has since kindly told me that this statement is correct.

[3] Quoted in *The Monist*, Vol. XX, p. 50.

cerned with the truth or otherwise of our hypotheses or consequents, but merely with the truth of the deductions.

The import of another alteration I quite fail to grasp. Mr. Russell wrote that "pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing." The *International Monthly*[4] put "asseverations" for "assertions"; and so Dr. Carus[5] remarked: "I wish Professor Russell would not describe mathematics as consisting of 'asseverations'; the very idea is jarring on my conception of the nature of mathematics."

When Dr. Carus[6] uses here, as he often has before, the word "anyness" to describe what is the fundamental characteristic of mathematics in his conception, he seems to be in agreement with one of the main tenets of Mr. Russell:[7] the propositions of logic "can be put into a form in which they apply to anything whatever"; "we never know what [which thing] we are talking about" in mathematics; the assertions are that, "if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing."

I am going to try shortly to explain Mr. Russell to my readers. Mr. Russell's work on the principles of mathematics and the relation of mathematics to logic "is by no means," as Couturat said,[8] "like certain philosophical systems in fashion, a brilliant paradox, an individual and ephemeral fantasy, without roots in the past and without fruits in the future, but the necessary culmination and crowning of all the critical researches to which some mathematicians have given themselves up for the last half-century. It is a well-known fact that modern mathematics have constantly tended to deductive rigor of the reasonings and logical purity of the concepts. To these new needs of the scientific spirit a logic more and more exact and refined had to respond; the indispensable instrument of this new logic is the 'symbolic[9] logic' invented by Peano, practised by a whole school of mathematicians, and perfected by Russell.

[4] Quoted in *The Monist*, Vol. XX, p. 50.

[5] *Ibid.*, p. 53.

[6] *Ibid.*, p. 50.

[7] *Ibid.*, pp. 47, 49, 50.

[8] *Les Principes des mathématiques*, Paris, 1905, pp. v-vi. A translation of Couturat's work by the author of this article is in preparation.

[9] As a matter of fact, Peano has always called his system "mathematical logic." The name of Frege ought to be mentioned with Peano's in this connection.

It is owing to this *logistics* (as we will call it) that all mathematical theories have become susceptible of being subjected to a precise and subtle analysis, and of being reconstructed logically with a small number of fundamental data (primitive principles and notions). It is owing to this that Russell has been able, while completing on certain points this work of logical reduction, to systematize all the results acquired in a vast and profound synthesis, which is the quintessence of preceding works, and which manifests the spirit of modern mathematics."

Consider, for a moment, what this logical analysis means. Take the science of arithmetic. All its material and principles have to be reduced to logical terms and expressed unambiguously. This enormously important work is extraordinarily long and often tedious. Processes of thought that most mathematicians perform more or less accurately by "intuition" often take up, in expression, pages of symbols of logical deduction—if such deduction is possible; but then we get complete, and not only "moral," certainty, and an insight into the structure of certain truths. In Dr. Whitehead and Mr. Russell's latest book[10] there are 666 pages, most of them written in symbols, often with abbreviated proofs, and yet the definition of numbers is not yet reached! Things called "1" and "2" are defined, but not till the second volume will it appear that they are numbers!

There is a story current in Cambridge that, after a term's lecturing on the principles of mathematics, Mr. Russell informed his hearers that if they were good they should do simple addition next term....And so recently as 1888 Dedekind's tract of 58 pages, *Was sind und was sollen die Zahlen?*[11] was derided by some mathematicians because it devoted so much space to the foundations of arithmetic!

Few people can see the immense importance of Mr. Russell's work; fewer know how laborious it has been and by what splendid qualities of mind and character it has been inspired. That is all I can say on this head, as I do not wish to gush and am not writing an obituary notice. Not quite so few people know how brilliant Mr. Russell's work is. Mr. Russell's investigations have revealed some very striking things, and Mr. Russell has said them strikingly —said them, too, in books and articles which are read with delight, and sometimes with profit, by those who are untrained to follow

[10] *Principia Mathematica*, Vol. I, Cambridge, 1910.

[11] English translation by W. W. Beman, in Dedekind's *Essays on the Theory of Numbers*, Chicago, 1901.

Mr. Russell's work. I suppose Mr. Russell has a natural love of paradox, but his paradox is always used to give point to the statement of some truth. In his talk and writings, Mr. Russell is conscientious, truth-loving, keen and witty.

I now propose to analyze the *International Monthly* article and to try to show how the fundamental doctrines of the *Principles of Mathematics* are shortly stated in it. This will continue my article in *The Monist* for January, 1910;[12] and in future I hope to trace Mr. Russell's work beyond 1903.

I.

The first published indication of the effect of Peano's work on Russell appeared in an article by Russell on "Recent Work on the Principles of Mathematics" in the *International Monthly* for 1901.[13] Boole, he said,[14] was "mistaken in supposing that he was dealing with the laws of thought: the question how people actually think was quite irrelevant to him,.... His book was in fact concerned with formal logic, and this is the same thing as mathematics." Then came[15] a definition of pure mathematics: "Pure mathematics consists entirely of assertions to the effect that if such and such a proposition is true of *anything*, then such and such a proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it is supposed to be true. Both these points would belong to applied mathematics. We start, in pure mathematics, from certain rules of inference, by which we can infer that *if* one proposition is true, then so is some other proposition. These rules of inference constitute the principles of formal logic. We then take any hypothesis that seems amusing, and deduce its consequences. *If* our hypothesis is about *anything*, and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."

The reduction of mathematics to logic was spoken of:[16] "Now the fact is that, though there are indefinables and indemonstrables in every branch of applied mathematics, there are none in pure

[12] Vol. XX, pp. 93-118.
[13] Vol. IV, pp. 83-101.
[14] *Ibid.*, p. 83.
[15] *Ibid.*, pp. 83-84. For "assertions" was misprinted "asseverations," and for "amusing" was misprinted "assuring."
[16] *Ibid.*, p. 84.

mathematics except such as belong to general logic. Logic, broadly speaking, is distinguished by the fact that its propositions can be put into a form in which they apply to anything whatever. All pure mathematics—arithmetic, analysis, and geometry—is built up by combinations of the primitive ideas of logic, and its propositions are deduced from the general axioms of logic, such as the syllogism and the other rules of inference."

When dealing with questions of the principles of mathematics, the function of symbolism is exactly the opposite to that of symbolism in the other parts of mathematics. Russell said:[17] "The fact is that symbolism is useful because it makes things difficult. (This is not true of the advanced parts of mathematics, but only of the beginnings.) What we wish to know is, what can be deduced from what. Now, in the beginnings, everything is self-evident; and it is very hard to see whether one self-evident proposition follows from another or not. Obviousness is always the enemy of correctness. Hence we invent some new and difficult symbolism, in which nothing seems obvious. Then we set up certain rules for operating on the symbols, and the whole thing becomes mechanical. In this way we find out what must be taken as premise and what can be demonstrated or defined."

II.

Referring to Peano's three indefinables in arithmetic, Russell remarked:[18] "Even these three can be explained by means of the notions of *relation* and *class*; but this requires the logic of relations which Professor Peano has never taken up."

Russell[19] then indicated his contradiction:

"There is a greatest of all infinite [cardinal] numbers, which is the number of all things altogether, of every sort and kind. It is obvious that there cannot be a greater number than this, because, if everything has been taken, there is nothing left to add. Cantor has a proof that there is no greater number, and if this proof were valid, the contradictions of infinity would re-appear in a sublimated form. But on this one point, the master has been guilty of a very subtle fallacy, which I hope to explain in some future work."

* * *

Russell's statement of Zeno's puzzle about Achilles and the tortoise was:[20]

[17] *Ibid.*, pp. 85-86. [18] *Ibid.*, p. 87. [19] *Ibid.*, p. 95.
[20] *Ibid.*, pp. 95-96.

"The argument is this: Let Achilles and the tortoise start along a road at the same time, the tortoise (as is only fair) being allowed a handicap. Let Achilles go twice as fast as the tortoise, or ten times or a hundred times as fast. Then he will never reach the tortoise. For at every moment the tortoise is somewhere, and Achilles is somewhere; and neither is ever twice in the same place while the race is going on. Thus the tortoise goes to just as many places as Achilles does, because each is in one place at one moment, and in another at any other moment. But if Achilles were to catch up with the tortoise the places where the tortoise would have been would be only part of the places where Achilles would have been. Here, we must suppose, Zeno appealed to the maxim that the whole has more terms than the part. Thus, if Achilles were to overtake the tortoise, he would have been in more places than the tortoise; but we saw that he must, in any period, be in exactly as many places as the tortoise. Hence we infer that he can never catch the tortoise. This argument is strictly correct if we allow the axiom that the whole has more terms than the part. As the conclusion is absurd, the axiom must be rejected, and then all goes well. But there is no good word to be said for the philosophers of the past two thousand years and more, who have all allowed the axiom and denied the conclusion."

* * *

The converse of the Achilles, which Russell called "the paradox of Tristram Shandy," was then described;[21] and the remark was made[22]that the notion of continuity depends upon that of *order*, and that "nowadays, quantity is banished altogether [from mathematics] except from one little corner of geometry, while order more and more reigns supreme." Nowadays, too, a limit is defined ordinally.[23]

Then:[24] "Geometry, like arithmetic, has been subsumed in recent times under the general study of order. It was formerly supposed that geometry was the study of the nature of the space in which we live, and accordingly it was urged by those who held that what exists can only be known empirically, that geometry should really be regarded as belonging to applied mathematics. But it has gradually appeared, by the increase of non-Euclidean systems, that geometry throws no more light upon the nature of space than arithmetic

[21] *Ibid.*, pp. 96-97.

[22] *Ibid.*, p. 97.

[23] *Ibid.*, pp. 97-98.

[24] *Ibid.*, p. 98.

throws upon the population of the United States. Geometry is a whole collection of deductive sciences based on a corresponding collection of sets of axioms. One set of axioms is Euclid's; other equally good sets of axioms lead to other results. Whether Euclid's axioms are true, is a question as to which the pure mathematician is indifferent; and what is more, it is a question which it is theoretically impossible to answer with certainty in the affirmative. It might possibly be shown, by very careful measurements, that Euclid's axioms are false; but no measurements could ever assure us (owing to the errors of observation) that they are exactly true. Thus the geometer leaves to the man of science to decide, as best he may, what axioms are most nearly true in the actual world. The geometer takes any set of axioms that seem interesting, and deduces their consequences. What defines geometry, in this sense, is that the axioms must give rise to a series of more than one dimension. And it is thus that geometry becomes a department in the study of order."

Russell[25] then shortly dealt with the methods used by Peano and Fano in geometry, and finally[26] remarked that "the proof that all pure mathematics, including geometry, is nothing but formal logic, is a fatal blow to the Kantian philosophy."

III.

Let us now point out how this popular article gives indications of his logical work up to 1903.

To begin with, the two great influences on Russell's mathematical and logical work were Georg Cantor and Peano. Cantor had, in 1895 and 1897,[27] brought his researches on transfinite numbers and ordinal types to a close by two articles in which the principles of the subject were stated in an almost perfect logical form. Obviously, the whole question threw a great and welcome light on the principles of arithmetic.[28] Peano invented a symbolic logic which was especially adapted to the analysis and expression of mathematical theories. But Peano's logic was incomplete. It neglected the logic of relations, which was founded and developed by De Morgan, C. S. Peirce, and Schröder; and only contained a symbolical expression of the theory—unused, by the way, in Peano's symbolic

[25] *Ibid.*, pp. 99-100.

[26] *Ibid.*, p. 101.

[27] *Mathematische Annalen*, Vols. XLVI and XLIX. An annotated translation of these articles by the author is in preparation.

[28] Cf. my article on "Transfinite Numbers and the Principles of Mathematics" in *The Monist* for January, 1910.

exposition of arithmetic—of the "representations" of Richard Dedekind.[29] The logic of relations was, as Schröder had observed, necessary for the translation of Cantor's conceptions and proofs into a symbolic (speaking technically) form; and it was necessary in order to complete Peano's theory of arithmetic by defining in logical terms the three indefinables referred to above. Russell completed Peano's logic by a logic of relations in which the Peirce-Schröder ideas were modified so as to fit in with a logic which comprised more subtle distinctions than that of Schröder, in two papers, "Sur la logique des relations, avec des applications à la théorie des séries," and "Théorie des séries bien-ordonnées," which were published in Peano's *Revue de Mathématiques* for 1902,[30] and of the first of which an account was given in Russell's *Principles of Mathematics* of 1903.[31] The logic of relations gave to Russell the means of defining Peano's indefinables of arithmetic, and of proving his primitive propositions of arithmetic.[32]

Peano had emphasized that it was the notion of implication between propositions containing variables—or, as Russell expressed it, of *formal* implications[33] between propositional *functions*,[34] and not implication between (constant) propositions, that is used in mathematics. Further, the development of non-Euclidean geometry had shown in the most striking manner that, in pure mathematics, as in formal logic, we are not concerned with the truth or otherwise of the hypotheses. "Until the nineteenth century," said Russell,[35] "geometry meant Euclidean geometry, *i. e.*, a certain system of propositions deduced from premises supposed to describe the space in which we live....," but now, owing to investigations with premises other than Euclid's, "geometry has become....a subject in which the assertions are that such and such consequences follow from such and such premises, not that entities such as the premises describe actually exist." And all this goes some way

[29] Cf. the English translation of Dedekind's pamphlet in Dedekind's *Essays on the Theory of Numbers*, Chicago, 1901.

[30] An account of Peano's and Russell's logic was given by A. N. Whitehead in his paper "On Cardinal Numbers" in the *Amer. Journal of Math.*, Vol. XXIV, 1902, pp. 367-394.

[31] *The Principles of Mathematics*, Vol. I [the *Principia Mathematica* of Whitehead and Russell, of which the first volume was published in 1910, takes the place of the second volume], pp. 23-26; cf. Couturat, *op. cit.*, pp. 27-34.

[32] *Principles*, pp. 124-128.

[33] *Ibid.*, pp. 5, 11, 14, 36-41; Couturat, *op. cit.*, pp. 4, 21.

[34] *Principles*, pp. 13, 19; Couturat, *op. cit.*, p. 17.

[35] *Principles*, pp. 372-373.

towards explaining the definition of pure mathematics with which Russell's book begins:

"Pure mathematics is the class of all propositions of the form 'p implies q,' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants. And logical constants are all notions definable in terms of the following: Implication, the relation of a term to a class of which it is a member, the notion of *such that*, the notion of relation, and such further notions as may be involved in the general notion of propositions of the above form. In addition to these, mathematics *uses* a notion which is not a constituent of the propositions which it considers, namely the notion of truth."

In this definition culminates the discovery contributed to by Leibniz, Frege, Dedekind, Schröder, and a host of others, that pure mathematics is logic and logic alone. Hence Russell's[36] anti-Kantianism.

* * *

In the question of infinity, we have a discussion of Zeno's puzzles,[37] and meet again the paradox of Tristram Shandy.[38] When discussing continuity, Russell[39] made more explicit Cantor's discovery (1895) that it is a purely ordinal notion; and then, too, Russell succeeded in maintaining his theses that the theory of limits is purely ordinal,[40] that geometry is the study of order,[41] and that the notion of quantity is superfluous in mathematics.[42]

* * *

Finally we come to Russell's[43] contradiction. Starting from a study of Cantor's proof of 1892 that there is no greatest cardinal number, Russell discovered a very simple argument: If w denotes the class of all those entities x such that x is not a member of x; then, obviously, if w is a member of w, w is not a member of w, while if w is not a member of w, w is a member of w. This contra-

[36] *Principles*, pp. 4, 158, 259, 373, 442, 456-461; Couturat, *op. cit.*, pp. 235-308.

[37] *Principles*, pp. 347-353, 358-360.

[38] *Ibid.*, pp. 358-360.

[39] *Ibid.*, pp. 296-303; Couturat, *op. cit.*, pp. 91-97.

[40] *Principles*, pp. 276-277.

[41] *Ibid.*, p. 372; Couturat, *op. cit.*, p. 134.

[42] *Principles*, p. 158; Couturat, *op. cit.*, p. 98.

[43] *Principles*, pp. 364-368, 101-107.

diction, which threw doubt upon the legitimacy of the concept of class, and hence upon that of the science of arithmetic, showed itself as allied in principle to the paradoxes in the theory of aggregates discovered by Burali-Forti, König, Richard, and others, and to the old logical difficulty about the Cretan who said that Cretans were liars, and was only satisfactorily solved by Russell in 1905. Of this more elsewhere.

It only remains at present to refer to the work of Frege. He did his magnificent work on the principles of logic and mathematics alone and almost too independently, and his subtle distinctions and acute analysis have had great influence on modern work. But at first Russell had hardly heard of him, and re-discovered for himself many of his distinctions and views. In his *Principles*,[44] Russell devoted many pages to a careful critical estimate of Frege's work. I hope to give an account of Frege's work later.

PHILIP E. B. JOURDAIN.

THE LODGE, GIRTON, CAMBRIDGE, ENGLAND.

ALFRED BINET.*

OBITUARY.

Readers of *The Monist* are well acquainted with the name of Alfred Binet. That eminent psychologist died at Paris October 18, 1911, at the age of 54, from an attack of cerebral apoplexy. He was born at Nice, July 11, 1857. He first took up the study of law, but later turned his attention to natural sciences, and finally directed all his efforts to psychology. In 1894 in collaboration with Beaunis at the laboratory of physiological psychology of the Sorbonne, he founded the *Année psychologique*, an important publication of permanent value.

His principle works are *Vie psychique des micro-organismes* (English edition, *The Psychic Life of Micro-Organisms*, Open Court Publishing Co., 1894); *Psychologie du raisonnement* (English edition, *The Psychology of Reasoning*, Open Court Publishing Co., 1899); *Le magnetisme animal, Les alterations de la personnalité, Psychologie des grands calculateurs et joueurs d'échechs, Etude experimentale de l'intelligence, L'âme et le corps.* To these we should also add a number of articles on an equal variety of subjects, capil-

[44] Pp. 501-522.

* Translated for *The Monist.*

lary circulation, the pulse, emotions, character, graphology, the mystery of painting, etc.

In his last years he was particularly interested in the "psychological study of the child" and for this purpose founded a society which bore this title. In collaboration with Dr. Simon he published a number of studies on abnormal children. Very recently he suggested a system of "measurement of the development of intelligence in children" which seemed very simple and practical and has been tested by educators in many countries.

Simply to read the list of books and articles published by Binet might give the impression of too great a dispersion of forces. It is further true that the work of Binet does not, like that of other psychologists, present the development of one dominant thought pursued through all the problems of psychology. Nevertheless his work shows a unity of quite another kind, a unity of method. Binet always endeavored to apply the processes of experimentation or direct observation to the most diverse questions, and consequently we may say that inasmuch as his works tended towards the control or invention of facts, they form an important whole and bear constant witness to a truly scientific spirit. Although he did not conceive any broad hypotheses and did not aim at extended or conclusive solutions he was a prudent investigator of broad culture, rich and versatile intelligence and an excellent worker.

LUCIEN ARRÉAT.

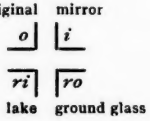
PARIS, FRANCE.

MAGIC SQUARES BY REVERSION.

The present number of *The Monist* contains an article on magic squares by Dr. C. Planck entitled "The Method of Reversion." This reminds the Editor of his own contributions to the problem of the construction of magic squares which appeared in Mr. W. S. Andrews's book on *Magic Squares and Cubes* under the title, "Reflections on Magic Squares."

Since these reflections were written I have come to the conclusion that a popular name for the several arrangements of the numbers in their cells would help greatly to make the idea clearer. On page 115 I have called the ordinary order *o*, the reversed ordinary *ro*, the inverse of the ordinary arrangement *i*, and by *ri* is understood the reversed inverted order. Considering the fact that all these

arrangements are brought about by a system of inversion which corresponds closely to reading the figures off in mirror writing, we may consider them as originated by placing a mirror on two sides of the original square. If *o* is flanked by a mirror from the top to the bottom it produces the order *i*. If the mirror is placed at the bottom it produces the order *ri* which mirrors the picture as if reflected in the surface of a lake, while the order *ro* lies in the corner between the two mirrors, being the reflection of either mirror in the other and this double inversion which we have called *ro* corresponds directly with the picture which appears on the ground glass of a photographer's camera. Accordingly the several orders on a plane surface might popularly be called the "original," the "mirror" reflection, the "lake" reflection and the "ground glass" picture.



FOUR WAYS OF INVERSION IN A PLANE.

Of course the conditions of such reflections grow more complicated if we venture from the plane into tridimensional space, and it can be extended into 4- and *n*-dimensional spaces. It appears to me that this idea of inversion rests ultimately on the same basis as Dr. Planck's method of reversions. P. C.